

What Is Proof?

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MAN'S THINKING flows in many channels and uses many techniques. He can gain new ideas by observing and investigating facts and data of all sorts and by drawing inferences from them. He can apply the facts and generalizations he has acquired to solving new problems, to explaining new events and phenomena.

These are methods by which he can gain new ideas on his own steam, so to say. A large part of his knowledge, however, comes from becoming familiar with the ideas already formulated by a multitude of other men. It is with reference to this mode of acquiring ideas that the understanding of the nature of proof and the skill in handling the processes of proof have special pertinence.

The concept of proof itself has many complex ramifications which it is impossible to discuss in the brief space of this article. The discussion here is limited to a few simple aspects of this

concept which are useful from the standpoint of teaching critical thinking. It should be sufficient to point out that, as an educational objective, the idea of nature of proof has broken the bounds of demonstrative geometry with its emphasis on formal processes of deduction. Today, the concept has been enlarged to include a variety of methods by which assurance about the validity of ideas and the acceptability of beliefs and opinions can be gained and communicated.

The present emphasis on training in the processes of critical thinking has been stimulated by several factors. First, a gradual accumulation of research data has brought a realization that critical thinking cannot be acquired as an automatic by-product of a mastery of other men's ideas and opinions, or of mastery of facts and information—an assumption which has dominated and still dominates education. It is now recognized rather generally that direct training in the processes of critical thinking is needed.

This is not a new idea. As early as 1790, John Locke pointed out the difficulty in acquiring real knowledge by absorbing uncritically other people's ideas:¹

The floating of other men's opinions in our brains makes us not one jot the

The unsettled state of the world today makes this an age of change—an age of questioning and weighing of values. At the same time, an abundance of propaganda—good and evil, alike—is being pressed upon us. If we are to make wise choices in the face of so many conflicting viewpoints, we must learn to reason clearly and rationally. In this article Hilda Taba of the University of Chicago discusses the nature of proof as it relates to the teaching of critical thinking and reminds us that an important part of our job is to train boys and girls to think.

¹ Adamson, T. W., ed: *The Educational Writings of John Locke*, New York, Longmans, Green, 1912, pp. 8-9.

more knowing though they happen to be true. What in them was science, in us is but opinionatry, whilst we give up our assent only to revered names and do not, as they did, employ our own reason to understand those truths which gave them reputation.

A second factor is that we have ceased to believe that rigorous training in geometric proof alone is sufficient to develop critical thinking in other areas or on matters of daily living. Partly, the cause for this is found in the fact that the formal teaching of geometry is inadequate for developing either an understanding of the techniques of sound argument or a general ability to think deductively.² Largely, however, the difficulty is that learning to reason logically about abstract and neutral problems does not prepare people for ability to reason logically on matters which are either more complex or which involve feelings and opinions.³

Logical processes have strong competitors in our own emotional bias and prejudice. Reliable facts and sound knowledge and principles of logic are often immobilized or misused when applied in areas in which attitudes and feelings tend to interfere with rational processes. This interference is the strongest on matters about which it is most important to formulate opinions and ideas dispassionately, namely, in the realm of controversial social and human problems. Here, the techniques and skills of controlling emotions and prejudice, of limiting the influence of bias

are most needed and least successfully achieved under present teaching.

For this reason there is a strong feeling that critical thinking should be taught in connection with the problems and material, where such thinking is expected to be applied in life.

A third stimulating influence has been the growing sensitivity to propaganda, intentional and unintentional. We are constantly bombarded with "proofs" designed to convince us that we should act in a certain way, believe in certain things, or accept the truth of certain ideas.

Some of these arguments are honest efforts to convince us about defensible things in fairly defensible ways. Others represent what is commonly called propaganda, namely, efforts to "sell" ideas or beliefs under false guises, or swerve us to action under false pretenses. Recent refinements in both the techniques of propaganda and the means of communication have made educators aware of the need of "immunizing" our growing generation against propaganda by developing a critical awareness of the techniques of propagandist arguments. These efforts were reflected in the work of the Institute for Propaganda Analysis,⁴ and in the emphasis in schools on examining advertisements and propagandist writing in the daily press and magazines.

Tools for Thinking

It is now more or less generally accepted that understanding the "nature of proof" is an important tool for any thinking citizen. What, then, are the

² Commission on Secondary Curriculum, P.E.A.: *Mathematics in General Education*, New York, Appleton Century, 1940, pp. 187-191.

³ Fawcett, Harold P.: *The Nature of Proof*, The National Council of Teachers of Mathematics, 1938, p. 75.

⁴ *Propaganda, How to Recognize It and Deal with It*, New York, Institute for Propaganda Analysis, 1938.

elements of it that could and should be taught as a part of general education?

Most presentations of ideas or arguments are carried on with reference to some set of assumptions, in the light of some evidence, and through the process of establishing a logical relationship between these assumptions and the conclusions. Unless we have the disposition and skill to examine these assumptions, to check the evidence and facts, and are sensitive to the logic of arriving at conclusions, it is impossible for us to make an intelligent reaction to the ideas presented to us.

It seems, therefore, that the following may be suggested as specific tasks for the school.

"Cure Your Cold With Xenex"

1. *Students should learn to detect assumptions underlying a variety of arguments and learn to scrutinize their validity and acceptability.*

All human thinking is carried on with reference to some set of assumptions. These assumptions may be good or bad, tenable or untenable, relevant or irrelevant to the problem at hand. Whichever they are, the conclusions reached are the logical outcomes of the assumptions on which they depend, and are true only to the degree that these assumptions are true. Thus, for example, if someone says, "Buy Xenex to cure your cold," he assumes, first, that colds can be cured, second, that they can be cured by medicines like Xenex, and, finally, that among all medicines Xenex is the most effective for curing colds. Whether or not one should buy Xenex depends on how valid these three assumptions are.

The evaluation of assumptions is by

no means an easy task. They are not always readily discernible. In propagandist writing, there is even a deliberate effort to disguise the assumptions which may lead to a criticism or questioning. Often, also, it is difficult to draw a line between what is an assumption and what is an established fact. In social sciences, particularly, few facts are established as true beyond doubt. Many a historic generalization which students learn as fact is nothing more than inference involving assumptions about what society is like—how human beings behave, and what the fundamental causes are for a certain series of events. For example, many writers of social science texts take it for granted that man's economic behavior is determined by the profit motive. To what extent is this an assumption? to what extent an established fact?

However, whatever the difficulties in examining any particular set of assumptions, a critical reaction to ideas is possible only to the extent that the assumptions underlying them are brought out in the open, and can be subjected to examination. Without such skill, all hopes for developing critical and independent thinking come to naught, for, willy-nilly, we must either gullibly accept the ideas that come our way, or else resort to an indiscriminate cynicism and suspicion towards all ideas.

If You Cut Down the Trees on Your Uncle's Farm . . .

2. *Students must learn to appraise the quality of evidence.*

Without going into the intricacies of the criteria for truly scientific evidence, students can learn to apply a few simple criteria in evaluating the soundness of

evidence. One such criterion is the sufficiency of evidence for a given purpose. Is, for example, the fact that trees on your uncle's farm are cut down sufficient evidence to conclude that forest resources are being depleted in America? Is the fact that a WPA worker leaned on a shovel in your father's garden sufficient evidence to generalize about the laziness of all WPA workers? Yet, too commonly, one instance, one observation, or even a second-hand story of an observation is accepted as sufficient evidence to prove or disprove an idea.

Evidence must also be examined from the standpoint of the source from which it comes. If someone says that Chinese are dirty, that Americans are materialistic, or that Italians throw knives, it is important to find out where he obtained that impression and to appraise the reliability of that source.

Too often, it is impossible to check personally the validity of evidence. We have to depend on experts either because the matter is beyond our competence or because it is impossible for us to conduct our own investigations. In such cases, it is necessary to form some judgment of the competence and authority of the person or of the source. It makes a difference whether a statement about the integrity of public officials comes from the Dies Committee or from a person or an organization of assured integrity, with no axe to grind on the given problem or issue. Similarly, it is important to determine whether a person who voices opinions on politics has competence in these matters or whether his sole qualification rests on achievements, such as, his popularity as an air hero or a football champion.

Weed Out the "Weasel Words"

3. *Students should recognize the role of definitions and acquire skill in determining the correct meaning of crucial terms and words.*

Language is an imperfectly used tool in conveying ideas. We use many "weasel words," the meanings of which are vague and wobbly. There are also many terms, particularly among abstract concepts, such as, "democracy" and "capitalism," which encompass a large territory and are, therefore, used only in their partial meaning in any specific context. Then there are words and terms which are emotionally colored, and, therefore, convey different meanings to different people.

Furthermore, definitions often shift in the course of an argument or presentation. Clinching an argument by systematically shifting the meaning of the terms used is a favorite propaganda device, though this difficulty is by no means absent in arguments with no direct propagandist intentions.

Since the rightness or wrongness of many a conclusion depends on the meaning given to crucial words and on how well that meaning is maintained throughout the argument, it is important that students learn to locate crucial words and terms requiring definition, and to define them accurately.

This problem is more complex than locating "weasel words" and their dictionary definitions, or picking out words with a maximum of emotional appeal and a minimum of correct meaning. It is simple enough to pin down the meaning of words like "income tax" and "restaurant," which can be defined either by a concrete reference to the

thing itself, by a few factual data, or by a description. A much more difficult task is involved in classifying and checking the meaning of such terms as "frontier" or "government control." These terms are abstractions of a high order, and a perfunctory verbal definition is of little aid in establishing their correct meaning.

Assumptions to Conclusions

4. *Students should be able to isolate and to examine the structure of argumentation itself.*

It is, for example, possible to line up the main points of an argument and to see whether the conclusions logically follow from the assumptions and facts presented. In such a task, familiarity with a few basic principles of logical deduction is of help.⁵ However, since most materials in our daily reading and listening are far from being air-tight logical constructions, the main problem is to distinguish between ideas and facts which are relevant and those which are

⁵ Most commonly used in high school work is the principle of "If-Then" and that of Indirect Argument.

irrelevant to the points presented. Fallacious forms of argumentation, such as, attacking a person in order to discredit an idea, deliberate shifting of definition, or relying upon distortion by the use of emotionally charged words and references to put over a point, must be noted also.⁶

The emphasis in this article has been on the points dealing with criticism of ideas. In view of the prevailing emphasis on mastery of ideas, criticism is an important addition to the intellectual repertoire of our students. However, this tool can be easily misused, particularly with an overemphasis on analyzing materials obviously and intentionally distorted. Such training leads easily to cynicism towards any ideas, for rarely do we meet arguments with perfectly sound logic. It is, therefore, important to teach students to recognize constructive points in any argument, and to give them sufficient experience in developing and defending ideas of their own.

⁶ For a listing of the fallacious forms of argumentation see *Propaganda, How to Recognize It and Deal with It*. Institute for Propaganda Analysis.

Work Experience

From Paul R. Hanna of Stanford University, editor of *Educational Leadership's* department, "The Changing World," comes this one-sentence definition of the much-used term, **WORK EXPERIENCE**:

WORK EXPERIENCE involves absorbing youth into the normal economic and social activities of the home and community in such a manner that youth is given an individual responsibility for the performance of a job that shows immediate results in the production of goods or the rendition of services and is recognized by the community as making a contribution.

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