FOR MORE than thirty years, in New York City, a program of elementary school mathematics has been in the process of gradual evolution. This process has been given impetus in recent years by a program of intensive work designated here as “Operation CurDev-Math” (CURriculum in DEVelopmental MATHematics). Aspects of this program are operating at the present time in each of the public elementary schools of the city.

The present program builds upon the earlier work of such New York City leaders as Saul Badanes, Eugene Nife-necker, and others, which involved basic rethinking in the area of arithmetic, as well as reorganized remedial drill procedures. Others whose thinking has made contributions to this program are such specialists and researchers as Brownell, Clark, Grossnickle, Fehr, Kinsella, Lazar, Morton, Stern, Thiele, Young and others in the fields of mathematics and child development. The current research program is being carried on under the guidance of Laura K. Eads of the Bureau of Curriculum Research with the collaboration of many able supervisors and teachers.

The method of research used in this program is characterized as “Action” or “Developmental Research.” Such research seeks a synthesis of previous studies on a higher level of thinking. The application of this synthesis to the teaching and learning of mathematics under classroom conditions, the consideration of mathematics in its relation to the entire curriculum, and the growth and development of children, are major considerations. This requires the cooperation of many persons and involves their collaborating and sharing with respect to functions as supervisors, curriculum specialists, teachers of teachers, teachers of children, children in their classrooms, and parents.

FOUR PHASES OF THE PROGRAM

“Operation Cur-Dev-Math” has gone through, or is progressing through, the four phases which are outlined below:

PHASE 1: involves study, critical evaluation and synthesis of research in the field of mathematics. Effort was made to use techniques and devices previously discovered and invented in order to gain new insights into teaching and learning which would be applicable to a large school system with widely varying conditions of teaching and learning. One of the important starting points was the New York State “Sequential Learnings Chart” (New York State, Mathematics for Elementary Schools, 1938, p. 31).

This needed to be adapted so as to show interrelationships among the various processes. Thus a chart of “Interrelationships and Sequences of Numbers and Processes” (mimeographed 1946) was constructed and partly reproduced in an experimental bulletin, “Arithmetic: Kindergarten-Grade Three” (1947, out of print).

PHASE 2: involves tryout and experimentation in selected classrooms—
always alert to: (a) mathematics as a logical science and systematic sequence; and (b) the development of children's concepts. The techniques used have been a combination of applied research (applying principles already known) and action research (on-the-spot work with teachers and/or pupils devising and using techniques and procedures), both directed toward building a program consistent with premises laid down and criteria established.

In common with findings of other studies we have found that:
- The "non-mathematical" concepts that children have affect their mathematical learning.
- Children's “non-mathematical” and mathematical concepts are generally more vague and unformed than most teachers realize.
- Children's verbalizations in terms of expected learnings all too often are no assurance of their understanding.
- Much more time is needed to develop concepts than to memorize verbalizations.
- Mathematical learning is dependent upon the clarity of concepts.
- All children need adequate background of experiences which are as real and life-like as possible in order to facilitate concept development.
- All children need a thorough development of simpler concepts in order to learn new concepts.
- Each new concept requires a certain degree of maturity for its development (mental development is only part of such maturity).
- Children's learning is facilitated if they have opportunities to make discoveries, to find out, to get information, to "think through" solutions by themselves and if they have time to arrive at solutions in a variety of ways.
- Children need to see the sense in what they learn; that is, to get meaning. This involves learning to see things in relation to each other. It involves thinking about numbers and processes.
- Children arrive at meaningful generalizations when they have had many experiences, many opportunities to "think through" solutions, many opportunities to make estimations and approximations, opportunity to learn the nature of our mathematical concepts until meaning has become quite clear, continuously satisfactory mathematical experiences.
- Children's learning of processes is facilitated when they use numbers that are thoroughly comprehended by them. (Larger numbers are used by the children for many purposes, but not when processes are being developed.)
- Differences in learning characterize any group of children. Children can function effectively only when they understand what they are expected to do, when they can do their work successfully and thus find emotional satisfaction in it, when their practice is in meaningful situations so that it will last, when they can apply their mathematical learnings in social situations.

Coordination of the logic of mathematics and findings with respect to the development of children's concepts resulted in the preparation of a "Course of Study in Mathematics: Grades K-6" (New York City Board of Education, Brooklyn 2, New York, 1950. 20 p. 25$). This course of study presents an outline of basic principles with respect to learning in the area of mathematics, and a teaching program for children as they progress from one developmental level to another. Four major developmental levels have been identified: (a) engaging in experiences; (b) using representative materials; (c) thinking through of mathematical relationships; and (d) written computation.

Educational Leadership
Experimental tryout of course content is being conducted in pilot schools in advance of implementation throughout the city. The time schedule for implementation follows: Grade 1—1948-49; Grade 2—1949-50; Grade 3—1950-51; Grade 4—1951-52; Grade 5—1952-53 and Grade 6—1953-54.

Most of the materials for implementation are in mimeographed form at present, but will be incorporated in guides which will be available later.

PHASE 3: involves a teacher training program carried on by supervisors and by Developmental Mathematics teachers in each of the 23 districts of the city. Out of this program have come suggestions, evaluations and proposals which are contributing to the research program. Individual teachers, schools and districts are developing materials and procedures adapted to their own needs. In-service courses for teachers are a part of this program, as well as cooperative relationships with colleges preparing New York City teachers.

Helping teachers to gain an understanding of mathematics as a science, as well as of procedures for helping children learn more effectively is necessarily a gradual process. The development of experiences which are meaningful to children at varying developmental levels of learning, the use of mathematical materials organized to help children perceive mathematical relationships, the process of encouraging children to think and to use their own words as they derive mathematical concepts, the use of drill procedures which emphasize mathematical relationships rather than memorization without meanings, the development of written computation with meaning, the provision for individual differences, all require development far in advance of the days when memorization and recall (for test purposes) of number facts and techniques for computing or “problem solving” were the principal goals of classroom teaching.

PHASE 4: involves a program of continuous evaluation. This is particularly important since we are concerned with the development of a program which will result in better learning than has been possible with the methods of the past. We know that we are getting this in experimental classes, from the kind of thinking children in these classes are able to do and from the skills they have been able to develop. Teachers’ day-by-day evaluations and results of traditional standardized tests corroborate this.

A program of evaluation is being developed by the Bureau of Educational Research under the direction of J. Wayne Wrightstone, with the cooperation of the Bureau of Curriculum Research and the Division of Elementary Schools. Paper-and-pencil inventories designed to help the teacher determine children's mathematical concepts as well as mathematical skills are being developed. Observational techniques which can be used by the teacher to determine the mathematical growth of individual children in the classroom are also being devised and collated. To date, preparation, tryout and standardization of such materials for each of grades one through four are in progress.

Those who launch a program of this type must do so with the full knowledge that differences in any one class will be even greater than before. At the same time inventiveness and creativeness will increase: misconceptions and partial learning will be less. Both those pupils who are bright and those who are not as bright will benefit because they know, and know why they know.

—William H. Bristow, director, Bureau of Curriculum Research, Board of Education, City of New York.

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