terials in order to form the appropriate visual images. They develop the skillful discipline of refining their ideas in terms of the limitations of their materials. In this way, they achieve new levels of sensitive insight.

To help children grow in their capacity to explore their ideas in order to embody them in aesthetic form a teacher also leads them into contact with materials from our artistic heritage. They not only learn how other artists have created visual images for the interpretation of ideas, but they also realize their responsibility to seek clarity and directness in their own expression.

Through such learning, children come to value the quality of feelings and ideas. They grow in their understanding of themselves and of others and so achieve higher levels of emotional and intellectual maturity. This process of learning leads them to value the intrinsic satisfactions from probing, working and "playing" with ideas. They realize the spiritual quality of deeply felt feelings and ideas. They come to understand that the content of education through the arts is the creative process through which significant personal experience is filtered to derive meaningful ideas for refined and clarified expression.

BJARNE R. ULLSVIK

Basic Learnings in MATHEMATICS

This article suggests "behavior" descriptions of the basic learnings in mathematics. It emphasizes the creativeness of this science and points out our need for developing a more mathematically literate public to insure scientific inquiry and for maintaining a mathematical potential for scientific production.

MATHEMATICS has been described as "the mirror of civilization" and as the "handmaiden of the sciences." Both of these descriptive titles give basic characteristics of mathematics, because mathematics is a manifestation of man's divine gift of "responseability." As a mirror, mathematics describes man's achievements in solving problems, and thus contains his creative responses to new vistas created by previously solved problems. As a "handmaiden," mathematics has provided the spade work for the progress of the basic sciences, for mathematics serves both as the core and as the vehicle for challenging the frontiers. Without an adequate comprehension of mathematics, a student in pursuit of the sciences is irrevocably stymied.

It appears most obvious that our civilization is now confronted with problems that involve an increasing

knowledge of mathematics, and that our very survival is dependent upon their solution. These problems make one aware that in order to be free, our nation must be mathematically literate; and in order to be strong, our nation must be mathematically proficient. Being mathematically proficient and literate are necessary conditions for being strong and free, and recent history has shown that mathematical proficiency alone is not sufficient to insure freedom for scientific inquiry. Both the scientists and mathematicians need a literate public to protect their desire to pursue a new idea, just because it is a new idea.

Mathematical educators are confronted with problems of how to provide for mathematical literacy as part of general education, and also provide for a mathematical potential. Preparation for both of these objectives can not be delayed until college, or even high school, but must be a continuous development beginning in the primary grades. The answer does not lie solely in more and better mathematics, but rather in what constitutes the basic learnings of mathematics, and how those learnings can best be taught consistent with the American concept of education—that all the children of all the people will be given an optimum opportunity for education.

A poll conducted by *Fortune*[1] magazine indicated that mathematics along with English were both the best liked and yet the least liked subjects in the secondary school curriculum. Mathematics has other dichotomies, for certain concepts are indispensable to ambitions of some and expendable for others; some concepts are easy enough to be grasped by the remedial and others provide questions challenging enough to baffle the most gifted. Although mathematical concepts are developed without concern for practical values, almost all areas of living make use of these concepts as the machinery of practical application.

[1] *Fortune Magazine*, November and December 1942.

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New Theories of Learning

Even though the basic value of mathematics has never been questioned, mathematics has experienced some de-emphasis throughout grades one through twelve. The explanation for this de-emphasis does not lie so much in questions of what and how much to teach, but rather as a result of questioning the primary purpose of secondary education. This de-emphasis has had its toll, but also associated benefits. It is most doubtful if the prevalent more psychological organization, the more meaningful approach for understanding, and the wave of experimentation could have been achieved without the conscientious criticism from non-mathematicians, and the educators' demand for more concern for individual differences. The prepondrance of failures that characterized the teaching of mathematics only a few years ago could not be justified by any soul-salvaging concept of education. Thus, a change in the teaching of both elementary and sec-

ondary mathematics should have been welcomed.

Not all changes met with approval. In fact, there has been, and still is, a marked difference of opinion between educators and mathematicians as to the merits of the changes. Some of these changes were motivated by personnel from outside the classroom, for the public began to take seriously that the schools belonged to them. This provided a greater concern for the administration of education, and resulted in accepting that as a pupil advances in school, the better citizen he becomes; the better adjusted he will be; the more economically sufficient; and in general, the more effective contribution he will make as a participating member of our society. This faith resulted from an aggregate of personal experiences and publication of studies indicating an unjustifiable number of student drop-outs. Believing that such drop-outs resulted in a loss rather than a gain for our society, almost universal promotion now characterizes our elementary grades.

Regardless of how this "no flunk" policy can be criticized, basic research seems to indicate that greater benefits can be derived from maintaining pupils in school, rather than placing them in an environment where they tend to meet with less success and approval. Such a promotion procedure has given the high schools a different pupil personnel than characterized our schools a generation ago. Now about 80 per cent of all persons of high school age are enrolled in our high schools, and we deplore the fact that only about 50 per cent of those who enter continue to graduate. With this pervading concept of education, teachers must "learn them before they can teach them," and even the most conservative teacher must admit the unprofitable use of time and energy by requiring traditional first-year algebra of all entering freshmen.

New theories of learning have influenced the concept of basic learnings in mathematics. Learning is conceived as a "tying on process"; and unless this "tying on" takes place, parrot talk will usually result. This has motivated teachers to carefully analyze the learning difficulties and begin teaching at the existing level of understanding, rather than at the grade equivalent from which the pupil was promoted. Improved measures for recognition of individual differences have placed emphasis upon the significance of the learning taught for, because research indicates that the pupil will learn in the degree that the intended learning seems significant to him. This has placed much concern for group motivation, sectioning, double and triple tracks, use of applications meaningful to the learner, and more concern for teaching of basic concepts than practice and drill to acquire facility with moving symbols around.

Behavior Descriptions of Basic Learnings

The basic question confronting mathematical educators is the primary question of "what mathematics is of most worth?" This question has been answered on many occasions, but of course the answers vary from almost decade to decade, dependent upon the problems of society and learning trends in education. Perhaps the best known

and most widely accepted recent set of basic learnings in mathematics is that presented as a Check List.[2] This set of basic learnings consists of twenty-nine items, which are considered to be a must of every citizen by the Commission on Post-War Plans of the National Council of Teachers of Mathematics. Some college teachers of mathematics have stated that, if the entering freshman preparing for engineering and basic sciences would really understand all of these twenty-nine items, they would expect no more. Yet, the Commission designed the items primarily for general education.

There is always a danger that any set of basic learnings will serve as isolated objectives to be taught as ends; and thus, almost defeat the entire purpose of creating an abbreviated list of basic learnings. These twenty-nine items serve as an excellent check upon basic learnings in mathematics, and if used as the Commission intended, the Check List can render real service to those who evaluate present offerings against the needs of our society.

The following list represents an attempt to categorize a set of basic learnings in mathematics. These learnings essentially include the twenty-nine items on the Check List, but are described in terms of pupil behavior, hoping that a more general description will tend to remove vulnerability of being considered as isolated ends and provide descriptions more conducive to evaluation:

[2] "Guidance Report of the Commission on Post-War Plans," *The Mathematics Teacher*, Nov. 1947, 40: 315-339. Reprinted as *Guidance Pamphlet in Mathematics for High School Students*, The National Council of Teachers of Mathematics.

A pupil who has acquired basic mathematical learnings in—

I. Numerical Computation:

1. Performs the fundamental operations (including square root) with integers, fractions, decimals, and per cents with accuracy and reasonable speed.

2. Recognizes the cardinal, ordinal, place value, and one-to-one correspondence of number.

3. Estimates results prior to and performs an adequate check after computation.

4. Recognizes justifiable accuracy and precision of numerical result.

5. Recognizes the computational efficiency in use of a formula.

II. Symbolic Expression:

1. Simplifies a verbal quantitative relationship by use of symbols.

2. Chooses and maintains a clear meaning for symbols.

3. Distinguishes between symbols and their meaning.

4. Seeks and discovers new relationships through use of symbols.

5. Searches for the meaning of symbols in unfamiliar expressions.

6. Performs fundamental operations with symbols representing signed numbers.

III. Graphs and Tables:

1. Knows how to collect relevant data.

2. Knows how to choose and construct kind of graph or table for adequate representation.

3. Formulates conclusions justified by the data.

4. Recognizes some possible misinterpretations of graphs and tables.

5. Searches for trends, rapid changes, measures of central tendency and dispersion, and points of maximum, minimum, intersection, and inflection.

6. Knows how to interpolate and make justifiable extrapolation.

IV. Problem Solving:

1. Formulates the problem for possible solution.

2. Determines kind and scope of desired solution.

3. Recognizes the variables or factors affecting solution.

4. Collects, selects, and organizes data on variables affecting solution.

5. Decides and applies appropriate arithmetic or algebraic operations for solution.

6. Checks solution by internal consistency.

7. Searches for a new and more general solution.

V. Form and Space Perception:

1. Knows how to use the ruler, protractor, and compass in basic constructions and measurement.

2. Visualizes and differentiates between two and three dimensions.

3. Knows how to compute areas and volumes of commonly known two and three dimensional figures.

4. Uses the English and metric systems of measurement and converts from one system to the other.

5. Makes reasonable approximation of size of angles, areas, volumes, and distances.

6. Makes correct interpretation of distances and areas on scale drawings and maps in both English and metric systems.

7. Recognizes symmetry, similarity, congruence, and perspective in nature, constructions, drawings, and photographs.

8. Understands use of scale drawing and trigonometric ratios in indirect measurement.

VI. Nature of Proof:

1. Differentiates between the use of senses in an intuitive generalization and the deductive thinking characterizing a logical proof.

2. Understands the role of undefined terms, definitions, postulates, and previously proven conclusions in a logical development.

3. Accepts "truth" as an inescapable conclusion when valid thinking is applied to accepted assumptions.

4. Checks to determine if the converse of a given statement can be accepted.

5. Evaluates his conclusions and those of others by recognizing the implied assumptions accepted.

6. Recognizes the application of certain common non-sequitur forms of reasoning.

7. Recognizes and applies basic proven relationships between lines, angles, and distances in triangles, pairs of triangles, well-known polygons, and circles.

Promising Trends

The above "behavior" descriptions of basic mathematics are in obvious need of further clarification, but it is equally obvious that the descriptions go beyond a concept of mathematical understandings as consisting largely of facility with manipulative techniques in arithmetic, algebra, or geometry. The above descriptions place emphasis upon the creativeness of mathematics and accept as a basic premise that teaching is an efficient means of promoting self-discovery of ideas and solutions to meet the needs and ambitions of a contributing participant in our democratic society. Such a concept of teaching appears to be consistent with the present attempts of curriculum change in mathematics.

Are the new attempts actually providing a more mathematically literate public and creating a mathematical

potential sufficient to meet the present challenge—or threat? Unfortunately, opinions on this question are legion, and the research studies are not conclusive. A recent summary comparing our "typical" arithmetic pupils with those of the nineteenth century states:

"Today's child does as well on the processes in common use and a little better on thought problems. On processes that have become obsolete or have been postponed to later grades he does less well during the elementary-school period; he usually 'catches up' on the postponed items by the time he completes Grade IX." [3]

Few studies at the secondary level provide the necessary comparable data, but a real challenge in teaching of basic learnings in mathematics is provided by Beckman.[4] His study concludes that the twenty-nine competencies as described in the Check List are better taught in ninth grade general mathematics classes than in ninth grade algebra. Yet, the learning of these competencies was not especially significant in either organization of ninth grade mathematics.

Much is yet to be accomplished, especially at the secondary level. Most of the experimentation and course changes in mathematics have been designed for a more successful achievement of those who would previously have failed, and comparable thought has not been given to challenging and accelerating the education of the gifted. Let's give to the interested and able student "a Pullman ticket rather than a bill-of-lading," as expressed by Professor C .C. MacDuffee, of the University of Wisconsin. Few times in history has the salvation of all been so dependent upon great minds with a concern for "even the least," and the foreseeable future creates even a greater need for creating and perpetuating their kind. This challenge is of greater import to mathematics than to any other curricular area.

Present trends and experimentation provide much hope for the development of a sequence of elementary and secondary mathematics, which will more adequately prepare for a mathematically literate public to insure scientific inquiry, and the maintenance of a mathematical potential for scientific production. Such is the hope of the educators, the desire of the teachers of mathematics, the wish of the scientists, and the prayer of those who love freedom. Could there be a better base for cooperative effort to mutually develop a set of basic learnings for elementary and secondary mathematics?

[3] "Volleyed and Thundered," Frank W. Hubbard, *NEA Journal*, Oct. 1954, 43: page 399.

[4] "Is General Mathematics or Algebra Providing Greater Opportunity to Attain The Recommended Mathematical Competencies," M. W. Beckman, *Journal of Educ. Research*, Mar. 1954, page 509.