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Mathematics In Junior High School

At what maturity levels and with what materials can certain mathematical concepts be taught?

THE University of Maryland Mathematics Project (Junior High School), made possible by a grant from the Carnegie Corporation of New York, is in the third of its three years. The grant is administered by the College of Education, and policy is guided by an advisory committee, whose members represent the Department of Mathematics, the College of Engineering, and the Department of Psychology, as well as the College of Education. Members from outside the university represent the U.S. Office of Education, the Maryland State Department of Education, and the Maryland State Teachers Association, besides the four major public school systems near the university—Washington, D.C., Prince Georges and Montgomery Counties, Maryland, and Arlington County, Virginia.

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The primary goal of the project has been to determine maturity levels at which certain mathematical concepts can be appropriately taught, and to prepare course materials for a teaching sequence in grades seven and eight that is mathematically and psychologically sound, and appropriate to modern-day needs. Results of the study are to be made available to various persons and organizations with interest in curriculum research and planning.

Staff and Organization

Director of the project is John R. Mayor. The author joined the project at its inception, as associate director, and a second associate, Helen L. Garstens, joined the staff a year later.

From its beginning, UMMaP has involved classroom teachers, as well as mathematicians and experts in other fields. About 25 persons, recommended as competent teachers by their administrations, participated in a weekly seminar

on the University of Maryland campus. In this seminar they heard lectures on mathematics and psychology, and developed materials for preliminary trial in their own classes. On the basis of such experience, the writing teams prepared the experimental courses now being taught in grades seven and eight. The seminar has continued, and still holds a place of central importance to the project.

In addition to the director and associate directors, the staff now consists of three graduate assistants, a psychologist (part-time), who directs the evaluation of the experimental courses, and two secretaries. All but the secretaries have additional duties, such as the teaching of regular classes.

In the summer of 1958 the staff wrote the first part of the experimental seventh grade course, in addition to conducting a special four-week National Science Foundation Institute. The experimental course was taught in some 43 classes in the year 1958-59. It was at this time that a psychologist joined the staff to direct the evaluation. Experimental classes and control classes were selected, and extensive testing was done in all of these. Control classes were of two kinds: traditional classes taught by teachers also teaching experimental classes, and traditional classes taught by persons not familiar with the new course. At this writing, the data have been only partially processed, but an evaluation report is to be released in the near future.

In the summer of 1959 the seventh grade course was revised and the first part of an eighth grade course written. During 1959-60 the revised course is being taught in grade seven, and the new eighth grade course is being taught and will be evaluated. In this evaluation there are more kinds of control classes than the previous year, since the eighth grade course embraces much of traditional first-year algebra, as well as arithmetic. Therefore some control classes are algebra classes, and some are arithmetic classes. As in the previous year, some control classes are to be taught by experimental teachers and some by persons not familiar with the experimental materials.

Principles Followed

The content of both courses was chosen in such a way that several basic principles are followed. It was felt, for one thing, that the separation of algebra and arithmetic, which is usually made, is artificial and detrimental to learning. Therefore no distinction is ever made between the two subjects in the UMMaP courses. The word *algebra* never appears, even though the equivalent of first-year algebra, as traditionally taught, is embodied in the two years' work. Rather, *properties of numbers* are studied, and those properties used to carry out calculations, solutions of equations, factoring and the like.

It is also known to be important to seek unifying concepts, in order that the total number of basic ideas may be reduced. Unifying concepts of several kinds have been used in the courses. Among the most important of these is the concept of a *mathematical system*, which is introduced early and used again and again.

A third fundamental principle concerns the use of language. It soon becomes apparent to those concerned with the precise expression and communication of ideas that the language traditionally used in mathematics is heavily lacking in precision. Present vocabulary and habits of usage have been accumulated over thousands of years of mathematical activity, and one easily finds vague and

contradictory notions, as well as a lack of words with which to express today's mathematical ideas clearly and briefly. Therefore the writers have spent considerable effort in clarifying ideas by altering language and using it precisely.

A fourth fundamental principle is that the student should *understand* mathematics, rather than simply learn by rote a variety of manipulative skills. Furthermore, motivation and retention should be far greater when students can be led to make discoveries themselves. For these reasons the course materials, including texts and teacher manuals, have been written so as strongly to encourage discovery on the part of students. The discovery is to be both inductive and deductive, in accordance with good mathematical method. The deductive reasoning on the part of students is first very informal, becoming gradually more and more formal.

Progress to Date

No formal report on the experimental teaching can be made now, in the absence of statistical compilations. It is possible, however, to mention several items which indicate the progress of the project to date. For example, it has been demonstrated that junior high school children are far more capable than had been supposed. This may not be a surprise to many, but it is of foremost importance to demonstrate it conclusively.

Seventh graders, low average, average and above average, consistently grasped rather sophisticated mathematical ideas easily and with less difficulty than their own teachers. Excellent teachers reported again and again their surprise at how easily their students learned new concepts that had been difficult for the teachers themselves. Many experienced teachers, after teaching the experimental course, confessed that they had not believed it possible to teach such material to junior high school students. Apparently seventh grade children are mentally more mature than has been imagined, even by persons with good imagination, and their intense curiosity enables them to digest and enjoy mathematical concepts of some intricacy. Enthusiasm has been high.

Teachers have shown an increasing spirit and interest in the work. Administrators and parents have welcomed the new program, and the interest and enthusiasm of children have consistently been reported to be at a new high. The high spirit and enthusiasm of the teachers cooperating in the project have impressed the staff with the fact that it has probably conducted, incidentally, a new type of inservice program, so successful that it can be heartily recommended. The project has brought together classroom teachers, supervisors, mathematicians, psychologists and educators in a joint effort. The importance of this would be difficult to exaggerate. Relations between these various persons have been good, and the project could probably not have operated as it did otherwise.

Interest in the project and its experimental courses across the country and in foreign countries has grown rapidly. Cooperation has been given wherever possible to school systems, institutions of higher learning, and to other projects, including the School Mathematics Study Group. Samples of course materials have been made available at near cost to persons interested. These have included not only teachers and supervisors, but parents, professional mathematicians, and publishers. It can be expected that the writers of commercial textbooks will be influenced by these materials.

Research Implications

The implications of the work done so far are difficult to assess completely. It is possible, nevertheless, to draw some conclusions, based on the work of this, together with other projects. It is well agreed by persons concerned with the project that curriculum revision, other than that of a minor sort, cannot be done well by a single school system, even a large one, using present procedures. Rather, it is a task which needs the attention of research teams in the universities. Curriculum revision ought properly to be considered as an area of university research, with the cooperation of public and private schools. There must be freedom on the part of experimenters to try many kinds of innovations, without burden of the problems that ordinarily beset the classroom teacher in a school system. Moreover the experimenter must be relieved of any fear that his experiment may produce negative results.

In this way, research projects can determine what materials may be taught, and by what procedures, to students of various ability and maturity levels. Courses can then be written for possible adoption by school systems. It is important that a research team include specialists of several kinds. It is particularly important that the team include subject area specialists, and provision for liaison with subject area specialists, as well as specialists in psychology. Currently education is enjoying, in the areas of mathematics and science, cooperation between subject matter specialists and educators. Such cooperation has unfortunately been lacking in the past, and it is to be hoped that it will never again be lost, but can be increased, and spread to other areas, as time goes on.

It is obvious from the experience of several mathematics curriculum projects that no single best curriculum exists. Several experimental courses, with some similarities, but also with wide differences, have been written and tried, all with apparent success. The desire, seemingly universal, to standardize course content across the country, or within a school system, could well result in the eventual formulation of standard courses, but for continuing progress in curriculum development it would seem that such standardization should be avoided. Then the curriculum can remain open for gradual change, rather than becoming ossified and awaiting an eventual painful rejuvenation, such as is being experienced now.

Although the efforts of UMMaP are aimed primarily at grades seven and eight, implications for curriculum changes at levels below and above these became apparent early. Such is, perhaps, what should be expected. The view held by some, however, that curriculum research and building should begin in the early grades and work upward, is not tenable. Good research at any level will undoubtedly have implications for all levels. Here is a further point to emphasize that continuing experimentation and change should be embodied as a fundamental in the approach to curriculum development.

Since mathematics in the junior high school can, and apparently should be, far different than it has been for the past several decades, the need for new training programs for prospective teachers is clearly immense. Moreover there is a tremendous need for additional training for teachers in the field, and for revised certification requirements. The new training must necessarily include more and different mathematics content than have past and present curricula. Further-

more, the mathematics content must include contemporary approaches to mathematical concepts, whereas the traditional college mathematics courses do not.

It is not too early for teacher training institutions to consider a rapid change in their requirements and course offerings. Many have already begun. Nor is it too early for school systems to institute in-service training programs aimed at providing present teachers with an introduction to modern mathematical ideas, and at encouraging them to continue their study individually.

One universal result of research is the bringing of its shortcomings into focus, and showing the need for further work. The University of Maryland Mathematics Project is not exceptional in this regard. Several attacks on psychological problems have been made, with less success than had been hoped. Tape-recorded interviews of children were conducted, as well as special interviews of children by a trained interviewer, in attempts to discern readiness for concept formation, and to ascertain attitudinal development. Pressures of time have prevented the developing of these techniques, and the data have been useful to a more limited extent than would have been liked. A mathematics attitude inventory has been constructed, however, and is in process of being standardized and evaluated.

It is now realized that the problem of junior high school mathematics curriculum is so vast that the three years provided by the Carnegie Corporation grant is sufficient to provide only a good start. The staff agreed in the beginning to concentrate on grades seven and eight, for reasons of time limitation, and because the curriculum for these two grades is known to be the weakest link in the chain. In retrospect this appears to have been necessary, but not particularly wise. Curriculum workers, it appears, still are thinking in terms of the 8-4 plan, in spite of the fact that most school systems have gone, or plan to go, to the 6-3-3 plan. It appears to this writer extremely important to develop a *junior high school* curriculum, rather than an elementary and a secondary curriculum.

Research in mathematics curricula has now made it possible for rapid and drastic changes to be made, largely as a result of the financial support of the several foundations. Nevertheless, it is important for schools to plan for flexibility in curricula for a long time to come, and for universities to accept the financial responsibility for continuing the work which has been made possible by the assistance of the foundations.