John R. Mayor
and John A. Brown

New Mathematics in the Junior High School

Now being developed is a new approach to mathematics.

CURRICULUM development may be based on subject matter and on the psychology of problem solving and decision making. Studies in problem solving indicate that a clearly understood concept plays an important role in learning. In learning one may classify what he perceives through concepts already learned. Consequently, today's curriculum builders are interested in the psychology of concept formation.

Psychological Considerations

When a person is faced with a problem for which he does not have a ready solution, he seeks patterns and elements common to related problems for which he does have a solution. The information before him may be chaotic but he tries to make sense out of it. In trying to make sense out of a new situation, he looks for familiar patterns and principles or properties. Looking for a pattern is generally a matter of classification. Classification can be accomplished in two basic steps: (a) describing the class by common properties; or (b) listing all possible members of a class. In many cases classifying by properties will enable the learner to identify a configuration in a seemingly disorganized situation. Identification of such a scheme will facilitate (but not guarantee) a satisfactory solution of the problem. Further attempt to identify principles and properties as part of a familiar pattern may be necessary.

A curriculum in mathematics that provides experiences with mathematical structures and properties which are important in the development of new concepts enables the teacher to take full advantage of what he knows about concept formation and problem solving. For example, in the new junior high school mathematics a concept of a triangle, different from that possessed by Euclid, is introduced. This choice is made because the idea of triangle as the union of the sets of points on three line segments is more easily related to other mathematical structures which are a part of the course. A similar statement can be made about other ideas such as additive inverse and the number of selections of n things

John R. Mayor *is Director of Education, American Association for the Advancement of Science, Washington, D. C.* **John A. Brown** *is Professor of Education, University of Delaware, Newark.*

taken r at a time. Developing a concept is a matter of building upon past concepts and relating the old to the new. One of the most difficult problems in curriculum development is the blending of new ideas with old ideas to bring about a higher level of understanding.

Mathematical Considerations

The new mathematics courses for grades 7, 8, and 9 have been developed with first consideration given to mathematical properties. Thus there is emphasis on "systems." Properties of systems can be the basis of a classification method. The natural numbers are first introduced and such properties as groupings, order, closure, and identity are presented. After successful experiences with the natural numbers, the pupil then meets situations which cannot be solved by numbers in this set. The arithmetic numbers (whole numbers and fractions) are then identified and their properties are developed from experiences with natural numbers. The concept of number has thereby been extended to a higher level. When the time is appropriate, the concept of number is extended to the system of rational numbers, which includes negative numbers and zero. This new number system can be studied in terms of ideas already learned. An important point is that the properties identify the system.

Present psychological studies show that youngsters can learn many mathematical ideas which in the past we have reserved for adults. The ability of pupils to abstract and to do informal deduction has been underestimated. If a concept is studied in a proper setting many seemingly "difficult" ideas can be understood by the young learner. A curriculum problem is to select and present the appropriate setting. The vocabulary must be in the reach of the individual; there must be past experiences to build upon; new experiences must be built upon the old; and the new experiences must not be presented too rapidly.

New vocabulary is introduced to make ideas more precise and clear. The language of sets, which readily unifies ideas, is introduced and developed whenever the set language simplifies the discussion and enhances the attainment of a concept. The set concept may be used to show relationships with more familiar situations. For example, the term "set" is introduced and used for a collection of things; the terms "set," "intersection of sets," and "union of sets" are used with numbers and in geometry. You will recall the definition of an angle as a set of points. The terms "sentence" and "statement," whether used in arithmetic, algebra, or geometry, are used in precisely the same way in each application.

Goals and Processes

Close examination of the new mathematics courses for junior high school will show that, while the content and organization have been selected from the mathematical point of view, the choices are in keeping with sound learning principles. A proper setting for a new concept has been thoughtfully selected. In presenting an idea it is illustrated by concrete examples familiar to the pupil. By an appropriate arrangement of exercises, including exercises involving familiar ideas, the pupil may reach an abstract notion about the new idea. It is true that there is not just one way to do this and, therefore, the presentation must be examined and, if possible, improved. The new courses have gone through a period of try-out, which will be continued.

Mathematics for junior high school must be mathematically sound, psychologically presented, and contain information and skills for intelligent citizenship in our present and future society. High level mathematical competency is needed not only in scientific fields but also in the fields of humanities. The foreword in the School Mathematics Study Group's *Mathematics for Junior High School, Volume I*, states:

> The increasing contribution of mathematics to the culture of the modern world, as well as its importance as a vital part of scientific and humanistic education, has made it essential that the mathematics in our schools be both well selected and well taught.

The junior high school curriculum studies (and those at other levels) are the work of teams composed of mathematicians, classroom teachers, and psychologists. The mathematician supplies the mathematical ideas, the teachers the setting for teaching the pupils, and the psychologist furnishes guidelines for presentation and evaluation. This team approach is one of the most significant trends in curriculum development of this century. Team approach has long been recognized as sound but not always have financial resources been available to bring such an approach into being, certainly not on so broad a scale.

Some Evidence of Success

Examples [1] of important ideas and processes of curriculum development are selected from the courses for grades 7 and 8 of the School Mathematics Study Group, supported by the National Science Foundation, and of the University of Maryland Mathematics Project (Junior High School), supported by the Carnegie Corporation of New York. In the current school year the School Mathematics Study Group has in use by more than 100,000 students "sample" textbooks for grades 7, 9, 10, 11, and 12. These textbooks are revisions of preliminary editions which were tried out during the school year 1959-60 by more than 500 teachers. SMSG also has available a preliminary edition of a sample course for grade 8, much of which has had wide try-out as experimental units, and experimental units for grades 4, 5, and 6 which are being tried out during the current school year. The University of Maryland courses are also being widely used this year, after two years' experimental teaching. In all of the courses the mathematical approach has been used. The choices make possible application of what we know about learning.

Special reference has been made to the new mathematics for grades 7 and 8, partly because of the authors' closer association with the work at this level, but particularly because the new courses for these grades are a greater departure from traditional material. It seems clear that curriculum revision in mathematics is more greatly needed in these grades than in the others, as badly as it is also needed at other levels. In traditional mathematics for grades 7 and 8 emphasis has been on review, through drill, and motivation through the so-called social applications to budgeting, banking and insurance. In the new mathematics, review is provided in a new setting in which reasons for the "rules" of arithmetic become clear, and motivation is based on the innate curiosity of an in-

(Continued on page 189)

[1] More information on the SMSG and Maryland programs may be obtained by addressing inquiries to: School Mathematics Study Group, Drawer 2502A, Yale Station, New Haven, Connecticut; or to: University of Maryland Mathematics Project, Skinner Building, University of Maryland, College Park, Maryland.

creased opportunities for engaging in value clarification. In making an application to core programs one might inquire whether or not the longer block of time with a given teacher, the broader scope of problems, and the situation of interaction or of establishing relationships between or among fields of knowledge do contribute to greater *identification* or to increased *valuing* by the student.

Within these broad characteristics of a core program the investigator might locate a range of experimental situations which would offer useful comparisons. For example, an interesting comparison could be made between those programs where the scope is defined rather narrowly by the fusion of language arts and social studies and those incorporating problem areas of wider interdisciplinary relevance. Furthermore, it would be possible to study the significance of the extended block of time for instructional improvement.

While focusing its attention on the four areas mentioned, the Commission assumes that such programs will be evaluated continually along more traditional lines, that is, on their effectiveness in helping adolescents achieve competence in subject matter. Omission of such goals in the current study is premised on a belief that sufficient evidence has already been collected on this concern by numerous investigators. Reviews of recent research involving comparative studies of core and departmentalized programs of general education indicate rather clearly that the more experimental programs are quite satisfactory in this respect. In summarizing pertinent research, Alberty concluded that "few significant differences in student achievement as measured by objective tests have been found between students in core programs and those in other types of curricular organizations." [1]

Most studies suggest that the comparative advantages of a core program over more fragmented approaches are seen primarily in the realization of the behavioral goals of general education. The Commission hopes to mark out more distinctly the guide lines for evaluating core programs in these respects.

—VICTOR B. LAWHEAD, *Assistant Dean, Ball State Teachers College, Muncie, Indiana; and Chairman, ASCD Commission on Core Teaching*

[1] Harold Alberty. "Evaluation of Core Programs." *Encyclopedia of Educational Research*, Third Edition. New York: The Macmillan Company, 1960. p. 339.

New Mathematics

(Continued from page 167)

dividual about organizational patterns and the joy of understanding.

Testing of students who have completed the new courses shows that the students score quite satisfactorily on standardized tests of computation and reasoning based on traditional materials. Reports of teachers hold almost unanimously that the interest of students in the new mathematics is much greater than in the old. Too, it should not be interpreted that "new" neglects the social applications. The "social" is interpreted more broadly as, for example, in the introduction to probability and also the many more applications in the sciences.

The SMSG and Maryland 7th and 8th grade courses were written for all students in these grades. Experience with these courses to date seems to show that the same basic content and organization are appropriate for all ability and interest levels, when an appropriate adjustment is made so that those of lower ability and interest can cover the material more slowly.