A study of varied approaches to

Enriched Mathematics for Gifted Junior High School Students

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THE Cheltenham Mathematics Study grew out of an attempt to probe for a better understanding of *enrichment*. An analysis of teacher reports of experiences considered appropriate for gifted students, together with their explanations of "why," raised many more questions than it answered about the nature of enrichment. Members of the Cheltenham Gifted Youth Committee and the HMLI's Talented Youth Project staff discussed this analysis. They conceived a plan which led to designing a study of the effects on student achievement and attitudes of several approaches to the teaching of mathematics.

The plan involved studying "enrichment" in three different programs: (a) where concepts of modern mathematics were added to the existing courses of study; (b) where students were accelerated through the traditional mathematics sequence; and (c) where the focus was on more difficult and complex problems of the regular courses.

Incoming seventh graders in the fall of 1957 were selected and matched to build four comparable groups—three classes in one junior high school and the fourth in another. The criteria for matching students included I.Q. (within five points), arithmetic achievement (within six months), age (within six months), teacher rating (within three categories on rating scale), and sex. The comparability of the four groups can be seen in Table 1.

Four junior high school mathematics teachers were invited to participate and each was assigned to the experimental class of his choice. During the spring of 1957 and for the following school year, the teachers met regularly with the local coordinator and the HMLI staff members to plan the program sequences.[1] All four teachers met together to discuss various administrative and

[1] All four teachers met on two days with Professor Howard Fehr, head of the mathematics department of Teachers College, Columbia University, prior to the inception of the program. During the school year, 1957-58, the Enrichment teachers and other interested mathematics teachers not involved in the experiment met one evening each month for a voluntary seminar at which Dr. John Mauchly, co-inventor of UNIVAC, Remington Rand Corporation, Dr. Saul Gorn, head of UNIVAC Center, University of Pennsylvania; Mr. Leonard Starr, executive at the Michael Flynn Corporation; and Dr. Sid Axinn, professor of philosophy, Temple University, acted as consultants and helped in the development of supplemental units.

procedural questions. Questions of content, however, were treated separately with the Enrichment teachers and the Acceleration teacher in order to minimize their interaction and possible influence on each other's programs. Special mathematics bibliographies were prepared. Additional books and other instructional materials were purchased for both the school and the professional libraries. The specific program for each of the groups was carefully developed and weekly lesson plans were prepared by each of the teachers.

During grade seven (1957-58), the Accelerated Class covered the standard seventh and eighth grade arithmetic courses plus an introduction to elementary algebra. When, in early April, the Orleans Algebra Prognosis Test was administered, the results indicated that the class was ready to begin the 9th grade program. This was possible since the tempo of instruction was increased, review material curtailed, and some units drastically condensed. The two Enrichment Classes followed the standard seventh grade course modified somewhat to permit the addition of three enrichment units: History of Mathematics, History of Numbers, and Number Systems. The Control Class followed the standard seventh grade course, doing the more difficult problems in the regular text.

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In addition to the use of standardized tests of mathematical competence, the HMLI staff developed materials to assess attitudes toward mathematics and to test (a) interest span for working on mathematics problems (mathegram puzzles); (b) choice of a few difficult vs. many easy problems; (c) selection of computational or verbal problems of equal difficulty; and (d) choice of abstract or situational problems.

The attitude inventory showed no significant differences among the four groups at the outset of the study. The Davis Test of Mathematical Competence

Table 1.—Bases for Matching Students for Participation in Each of the Four Seventh-Grade Experimental Classes

Group	*N*	*Avg. I.Q.*	*Avg. Arith.*	*Avg. Reading*	*Teacher Rating*
Enriched-I	28	133	9.3	9.4	19 outstanding 9 good
Enriched-II	28	133	9.2	9.6	16 outstanding 11 good 1 fair
Accelerated	28	132	9.3	9.6	19 outstanding 8 good 1 fair
Control	28	133	9.3	9.8	15 outstanding 12 good 1 fair

Table 2.—Analysis of Variance of Raw Scores on STEP Mathematics Test Form 1-A of Four Mathematics Groups at the end of 9th Grade

Source of Variation	*Sum of Squares*	*d.f.*	*Mean Square*	*F*	*F.99*
Total	3488.88	103			
Among Means	585.30	3	195.10	6.72**	3.98
Within Groups	2903.58	100	29.04		

**Significant beyond the .01 level of confidence

was also administered early in the year. At the end of the year, all four groups took the mathematics part of the Stanford Achievement Battery and a teacher-made test of 30 items, 10 from each of the three approaches. The absence of test items on standardized instruments which would adequately assess the modern mathematics learnings necessitated the making of the teacher-made test, the purpose of which was to see how well the students in each class did on items to which they had not been directly exposed as well as on those items which they had studied.[2]

As an outgrowth of these experiences and after considerable study and discussion, the planning group decided to begin the University of Illinois Committee on School Mathematics program with one of the Enrichment Classes in the eighth grade. Preparation of the enrichment units dealing with modern mathematics concepts had stimulated examination and study of various programs including that of the UICSM. The availability of two Enrichment Classes permitted involving one of them in experimentation with still another approach to modern mathematics concepts. The Enrichment-I Class was designated as the Illinois Class for the following year. Its teacher attended a summer workshop and other meetings for orientation to the UICSM program and its materials.

[2] Comparison of the four groups on the standardized and teacher-made tests administered in the seventh and eighth grades will be reported in detail in another publication.

During the eighth grade (1958-59), the Accelerated Class completed the standard course in elementary algebra (normally a ninth grade course) and began Algebra II. The Enrichment Class completed the normal eighth grade arithmetic course modified to incorporate six enrichment units: Powers and Logs, Probability and Statistics, Topology, Computers, Permutations and Combination, and the Slide Rule. Most of these units were either developed by the teacher or adapted from existing materials. The Illinois Class completed Units I (Arithmetic of Real Number Systems) and II (Theorems and Generalizations) and began Unit III (Equations). The Control Class followed the standard eighth grade arithmetic program and, because it is part of the normal mathematics program in Cheltenham, started elementary algebra.

At the end of the eighth grade, all four groups took the STEP Mathematics Test, Form 2-A. Another teacher-made test consisting of 6 items from each of the four approaches was also administered.

In the ninth grade (1959-60), the Accelerated Class completed the Algebra II course normally taught to bright tenth graders. The Enriched Class completed the standard course in elementary algebra modified to permit teaching of three enrichment units: Laws of Arithmetic, Concepts of Inequality and Equation (a unit developed by the C.E.E.B. Commission on Mathematics), and

Logic. The Illinois Class finished the UICSM Unit III (Equations), Unit IV (Graphs), and Unit V (Relations and Functions). The Control Class completed the standard course in elementary algebra and, as is usual for bright pupils in Cheltenham, began the Algebra II course.

At the end of the ninth grade (May 1960), the STEP Mathematics Test, Form 1-A, was administered to all four groups. The publisher's college freshman norms were used in assessing percentile ranks. A 24-item test (again six items from each teacher) was given.

On the STEP Mathematics Test the Accelerated Class scored significantly higher than the Enriched and Control Classes (at or beyond the .05 level of confidence). Differences between the Accelerated and the Illinois Classes were not statistically significant. The Illinois Class scored significantly higher than the Control Class but did not do significantly better than the Enriched Class. The Enriched Class's mean score appeared somewhat higher than that of the Control Class, but the difference was not significant. On the teacher-made test, the Accelerated Class scored significantly higher than did either the Enriched or the Control groups (beyond the .05 level of confidence). The Illinois group scored significantly higher than the Control Class. Neither the difference between the Enriched and the Control Classes, nor the difference between the Accelerated and the Illinois classes, was significant. The data are presented in Tables 2 to 5.

A revision of the attitude inventory used in seventh grade was administered to all four groups. The items on the inventory dealt with the following topics: Mathematics Impact on Society, Characteristics of the Mathematician, Mathematics as a Career, The Nature of Mathematics, Self-Appraisal of Mathematical Ability, and The School's Effectiveness in Teaching Mathematics.[3] The four groups differed significantly on the number of "positive" or "correct" responses given in some of the categories but not in others. For most of the categories the order of the scores was similar to the pattern observed in the other tests—Accelerated and Illinois groups higher than the other two. (See Table 6).

During the ninth grade (1959-60), the senior high school mathematics teachers were actively involved in planning the further work of the four experimental groups, thus smoothing the way for articulation between the two school units during the second stage of the study. By the end of the tenth grade, the Accelerated Class will complete Plane and Solid Geometry. The Illinois Class will complete the UICSM Unit VI (Geometry) and the Third Course (Complex Numbers, Quadratic Equations, Exponents and Logarithms). There was some shifting of students between the Enriched and Control Classes. A few of the pupils in the Control Class joined the majority of the Enriched Class to work on the Algebra II course plus enrichment units as follows: Graphing Quadratics (from UICSM Unit V), Using Mathematics in Science (developed jointly by the science and mathematics teachers) and, if time permits, Matrices The remainder of the Control Class and a few members of the Enriched Class will complete the normal tenth-grade Algebra II program.

The finding that the two accelerated

[3] Some of the items were adapted from an attitude scale prepared by Hugh Allen, Jr., *Attitudes of Certain High School Seniors Toward Scientific Careers* (New York: Bureau of Publications, Teachers College, Columbia University, 1959, pages 47-50).

Table 3.—Comparison of Raw Scale Means of the Four Mathematics Groups on STEP Mathematics Test Form 1-A

Group	*Mean*	*Accelerated*	*Illinois*	*Enriched*	*Control*
Accelerated	29.00		1.62	4.86*	6.00*
Illinois	27.38			3.24	4.38*
Enriched	24.14				1.14*
Control	23.00				

*Significant at or beyond the .05 level of confidence using the Sheffé technique for the comparison of means

Table 4.—Analysis of Variance of Scores on "Teacher-Made" Test of Four Mathematics Groups at the end of 9th Grade

Source of Variation	*Sum of Squares*	*d.f.*	*Mean Square*	*F*	*F.99*
Total	1091.39	103			
Among Means	301.62	3	100.54	12.72**	3.98
Within Groups	789.77	100	7.90		

**Significant beyond the .01 level of confidence

Table 5.—Comparison of Means for the Four Mathematics Groups on "Teacher-Made" Test

Group	*Mean*	*Accelerated*	*Illinois*	*Enriched*	*Control*
Accelerated	10.74		1.03	3.17*	4.38*
Illinois	9.71			2.14	3.35*
Enriched	7.57				1.21
Control	6.36				

*Significant at or beyond the .05 level of confidence using the Sheffé technique for the comparison of means

Table 6.—Percentage of "Correct" Responses Given by Each of the Four Mathematics Groups in Each of Six Categories of an Attitude Inventory

Category	*Group*				
	Accelerated	*Illinois*	*Enriched*	*Control*	χ^2 (*using frequencies*)
1. Mathematics Impact on Society	49.0	62.5	59.8	53.6	2.05
2. Characteristics of the Mathematician	67.2	70.0	58.2	66.4	3.22
3. Mathematics as a Career	52.5	57.7	38.8	44.2	9.50*
4. The Nature of Mathematics	68.6	74.7	67.9	60.7	2.75
5. Self-Appraisal of Mathematical Ability	55.8	53.6	37.9	40.3	24.60*
6. The School's Effectiveness in Teaching Mathematics	68.0	63.2	58.7	57.1	2.15
7. Total Test	60.2	63.6	53.6	53.7	27.20*

*Significant beyond the .05 level of confidence $\left(\chi^2_{.95\ 3d.f} = 7.81\right)$

The Accelerated group did not differ significantly from the Illinois group; nor did the Enriched group differ significantly from the Control group. However, comparison of frequencies of "correct" responses for the Accelerated and Illinois groups combined as against the Enriched and Control groups combined, found significant differences in Categories 3, 5, 7. In each of these three categories the attitudes of the two combined accelerated groups, Accelerated and Illinois, were more positive than those of the other two groups combined.

groups—Accelerated and Illinois [4]—did consistently better on all the instruments used, suggests that the earlier introduction of more difficult content and/or the increased tempo of instruction result in greater mathematical competence and in somewhat more positive attitudes toward mathematics than do the other approaches used in this study. The differences between the Accelerated and Illinois groups were not statistically significant on any of the comparisons made, although the former more often scored higher than the latter. However, the fact that there was only one class of each kind and a separate teacher for each class necessarily limits the extent to which the results of the experiment can be generalized. The study is now being replicated in Cheltenham with four other classes.

In addition to providing information on the relative effectiveness of varied approaches to mathematics for gifted junior high students, this study also resulted in a better understanding of the meaning of enrichment, its evaluation, the role of the teacher in such an experimental program, and the impact of such a study on the school program.

1. The experiences throughout this study have made it clear that acceleration and enrichment are not opposing concepts. On the contrary, acceleration, either through the standard curriculum or through newly conceived curricula, provides students with meaningful and enriching experiences. Enrichment, on the other hand, becomes meaningful only when students deal with more advanced and more difficult concepts.

2. A perennial problem in assessing the results of an experiment which varies course content is that the usual measures are appropriate only where each of the groups is moving through the standard content at varying rates. They are inappropriate for assessing the effects of material such as the Illinois program or the Enrichment units. The construction of tests consisting of items from various programs results in measures which provide a fairer means of determining how well the students have learned the content unique to their own program and how well they can transfer these learnings to the application of concepts which had not been a part of their particular course content.

3. Involvement of the teachers in all phases of the study stimulated noticeable professional growth. Through their regular planning seminar sessions, through contact with specialists from the community who shared their insights into mathematics with them directly and with the students indirectly, through extended exposure to the literature of mathematics, the teachers became more competent in mathematics, in techniques of evaluation, and, to some extent, in research procedures. In addition, their confidence in themselves as professionals seems to have grown.

4. The study had an impact on the total mathematics curriculum for all students. As a result of the experience of the teacher of the Illinois Class, several other teachers now have undertaken the UICSM program and the Illinois First Course has become standard in the mathematics sequence in the Cheltenham junior high schools. Furthermore, greater articulation between the junior- and senior-high-school programs has resulted. Planning for sequence and continuity has emerged from cooperative efforts of the teachers at both levels.

[4] The Illinois Class was also accelerated since it began the UICSM program in grade eight, a year earlier than is usual.

Pupils are now scheduled into special groups for mathematics study in the high school on the basis of the ninth grade teachers' recommendations.

The design and execution of this study are relatively clear and can be replicated in other school systems. However, the following points should be considered:

1. Grouping gifted students for differentiated instruction in mathematics highlighted the problem of the relative prestige associated with the various approaches. Acceleration—which implied going faster and being ahead of the others—was perceived by the students as having a special mark of distinction. Perhaps this could have been obviated by not requesting parental approval for participation in the accelerated program —a procedure not required for membership in the other groups.

2. Placing gifted students in special groups does not eliminate the need to individualize instruction. Some intellectually gifted students needed remedial-type instruction, while others needed guidance in independent study and work skills. Eliminating many of the rigid expectations traditionally imposed by courses labeled seventh and eighth grade arithmetic and first and second year algebra, freed the student and the teacher. Selected individuals accelerated even beyond the Accelerated Class and, in a few instances, will be ready for the Advanced Placement Program by the end of tenth grade. The experiment has helped to reinforce the notion that gifted pupils *do* need to be taught and are dependent on teachers, whether the teacher be a regular faculty member or an individual in the community who can provide the knowledge and stimulation necessary for sustained achievement.

3. Grouping on the basis of intelligence and achievement data was generally satisfactory. However these data gave few clues to the pupils' motivation to learn mathematics. Grouping had to be flexible. Some pupils were shifted from one group to another or dropped from the experimental classes over the three year period. There was little change in the relative standing of the pupils during the course of the study. Those who scored comparatively low in the first year were still among the lowest at the end of the ninth year. Likewise, the students who scored highest in the first year of the study were still among the top scorers three years later.

In a subject such as mathematics, the study showed clearly that acceleration in content and instructional tempo yielded enriched learnings. To what extent the generalization applies to other subject areas in which the structure and sequence are less sharply delimited requires further experimentation and research.