

# The Old Mathematics in the New Curriculum

*Problem: to match sound mathematics and sound pedagogy.*

THE publicity that has accompanied many of the recent developments in mathematics education has been received with mixed feelings by teachers, administrators and school patrons. School people [and parents] who recall unhappy times in the mathematics classroom may believe that a new era is being prepared for students because the old incomprehensible mathematics is going to be replaced by new and useful mathematics—high school algebra which was so puzzling is now going to give way to something called 'Boolean Algebra' which is up-to-date twentieth century stuff.

The mathematics teacher of the "old school" is very uneasy because it is likely that he has not studied the "new mathematics." And, even if he takes a summer institute course bearing the title 'Modern Algebra,' he may be all the more concerned because he probably feels, and with some justice, that he would have a very difficult time teaching subject matter which he barely understood. Finally, some parents view the situation with

either joy or misgivings because they feel that they shall no longer be able to conduct tutorial sessions around the dining room table.

It is my contention that too much has been made of the allegedly new mathematics in the new curriculum. In looking at the content of the new programs for grades 9-12, I am impressed more by the attempt to organize the traditional subject matter along logical lines than by the inclusion of new subject matter.

## Elementary Algebra

This attempt is most striking in the elementary algebra portion of the curriculum. What has heretofore been a year spent on learning an assortment of isolated rules and techniques which are quickly forgotten or confused is now a year devoted to learning (preferably discovering) a few basic principles and considering some of the logical consequences of these principles. These logical consequences amount to the same old rules and techniques as before, but the students' reaction to them is vastly different.

Let us consider an example. Consider the pedagogical problem of teaching a

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student to solve a system of equations "algebraically." Suppose that the student faces the problem of solving the system of equations:

$$(1) 3x+4y=18$$

$$(2) 2x-5y=-11$$

In the conventional curriculum, he is *told* that the thing to do is to multiply the sides of each equation by certain numbers so that the resulting equations will have either the x-terms or the y-terms alike. Multiply (1) by 5 and (2) by 4:

$$(1) (3 \cdot 5)x + (4 \cdot 5)y = 18 \cdot 5$$

$$(2) (2 \cdot 4)x - (5 \cdot 4)y = -11 \cdot 4$$

Then, add the equations. This will eliminate one of the unknowns.

$$(3 \cdot 5 + 2 \cdot 4)x = 18 \cdot 5 - 11 \cdot 4$$

Solve the resulting equation:

$$23x = 46$$

$$x = 2$$

Substitute in either of the given equations to find the other unknown. By dint of a tremendous amount of practice, the student finally masters the technique. But, it remains pretty much of a mystery to him. The justification for the steps involved is that they produce the right answer or that "this is the way the book does it."

Now, let us see how this problem is handled in one of the new programs. For one thing, the problem usually appears as an application of some more general idea. In this case, the general idea is that an equation in two variables defines a set of pairs of numbers. [For example, some of the pairs in the set defined by the first equation are (0, 4.5), (1, 3.75), (2, 3), (4, 1.5) and (5, 0.75).] So, solving this system of equations amounts to finding the pair of numbers which is common to the two sets of pairs.

One way to do this is to graph the two equations. The point of intersection of the two graphs gives you the sought-for pair. [The conventional curriculum may

not have taught graphing prior to this topic.] The student graphs these equations and sees the common solution. Now, the student is asked to "add" the equations and graph the result. He gets:

$$(3) (3+2)x + (4-5)y = 18-11$$

and, when he graphs this equation, he *discovers* that its graph passes through the point of intersection of the graphs of the original equations. This is moderately surprising and he may wonder whether this is just an isolated event, or whether it would happen with any two equations of the type he is considering. He has the basic principles to enable him to prove that it would always happen. Next, the student is asked to take equation (1), multiply both sides by some nonzero number, and graph the resulting equation. Say he multiplies by 7:

$$(1') (3 \cdot 7)x + (4 \cdot 7)y = 18 \cdot 7$$

He knows from previous experience that the graph of (1') is the same as the graph of (1). [Moreover, he can prove this.] Take equation (2); multiply both sides by, say 6:

$$(2') (2 \cdot 6)x - (5 \cdot 6)y = -11 \cdot 6,$$

and graph. Once again, the graph of (2') is the same as the graph of (2). Now, add equations (1') and (2'):

$$(3') (3 \cdot 7 + 2 \cdot 6)x + (4 \cdot 7 - 5 \cdot 6)y = 18 \cdot 7 - 11 \cdot 6$$

and graph the result. As before, the graph of (3') will pass through the common point of the graphs of (1) and (2). We now have an important general result. Suppose that r and s are nonzero numbers, and the sides of (1) are multiplied by r and those of (2) are multiplied by s. If the resulting equations are added:

$$(\bullet) (3r+2s)x + (4r-5s)y = 18r-11s$$

the graph of this equation will pass through the common point of the graphs of (1) and (2). Moreover, the values of 'r' and 's' determine the "tilt" of the graph of this equation. If this graph were

a vertical line, all of its points would have the same first number as the common point. So, the problem now becomes one of finding those values of 'r' and 's' for which the graph of equation (•) is a vertical line. The student knows that any equation for which the y-term is missing has a vertical line as its graph. So, he seeks numbers r and s such that  $4r - 5s$  is 0. The numbers 5 and 4 will do nicely. [So will 10 and 8.] Then, (•) becomes:

$$(3 \cdot 5 + 2 \cdot 4)x = 18 \cdot 5 - 11 \cdot 4$$

$$\text{or:} \quad x = 2$$

Hence, each point on this vertical line has first number 2. Since the common point is also on the vertical, its first number is 2. The second number for the common point can be found either by substitution or by choosing values of 'r' and 's' which convert (•) into an equation whose x-term is missing.

The student is now permitted to develop the most efficient technique he can out of this explanation. It is pretty clear that he does not have to think about the graphs. The whole job boils down to choosing the correct multipliers, r and s.

### An Understanding Approach

Objections to the new programs should not center on questions concerning new subject matter. The real question is whether or not students should *understand* the skills which are taught in both the conventional and the new programs. This is not a trivial issue. If you decide on an understanding approach, the implications of this commitment are far-reaching. For example, the question of time becomes important. It requires only a few minutes in a conventional program to *tell* students how to solve systems of equations. It will take the better part of a class hour to use the other approach.

Moreover, the other approach requires a consistency of treatment prior to this point in the curriculum in order to develop in the student a taste for and a delight in logical explanations. And, once this taste is cultivated, your later courses should be modified to keep students from rebelling at inconsistencies. An understanding approach requires much more of the teacher in terms of preparation, especially if the textbook has not been written in the same spirit.

Finally, there is the terrible hazard of thinking that any approach which emphasizes logical explanations leads to understanding. If such were the case, logical mathematics courses for high school would be plentiful, for mathematicians do know all the correct explanations. High school mathematics holds no mysteries for mathematicians. However, the trick is to build a curriculum in which the mathematics is logically developed and to match this curriculum both to the present interests, needs and capacities of the learners, and to their future interests and needs. For example, it would not require much to take the conventional approach to the system-of-equations technique described above and give a brief but sound mathematical explanation which did not refer to graphs. But, such an explanation would not call for the discoveries inherent in the graphing approach and would not appeal as much to students with strong geometric intuition.

The recent developments in high school mathematics education have not been concerned with replacing old subject matter with new subject matter. The primary task has been that of finding a matching between sound mathematics and sound pedagogy. The job has just begun.

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