Hazel Lundberg

# Mathematics in the Elementary School

*Goal: insight into relations and development of skills.*

SURELY at no time has mathematical education seemed so challenging and so intriguing. The creation of new mathematics, together with programs to improve the instruction and content of the subject, can make teaching and learning lively and exciting. Mathematics, which has long been established in the school curriculum, both as a humanistic study and as a useful scientific discipline, now seeks to adjust its place in education to its growing importance in contemporary life. Basic problems in such an adjustment include defining the mathematics of the new programs and discovering means for more effective instruction.

College and university mathematicians, teachers of mathematics, experts in education, and representatives of science and technology are directing their attention to mathematics education in an attempt to provide better programs and to insure a greater proportion of students whose interest in mathematics will persist at advanced levels. Mathematical education is continuous, and programs for its improvement necessarily are concerned with the mathematics of the elementary school.

Both content and methods of instruction depend in a large measure upon the goals to be achieved. Significant goals of school mathematics are the development of insight into the relations which give mathematics its characteristic structure and the development of technical skills. The two go hand in hand—mathematical thinking is based upon the understanding of basic concepts and principles, and mastery of technical skills is necessary for the machinery of thinking. William A. Brownell of the University of California, in accord with both goals, cautions against too great an emphasis on the mathematics of the subject. Such an emphasis, he states, can result in an arithmetic too deeply concerned with the science of numbers and too little concerned with the ability of children or the social purposes of mathematics.[1]

The shifting of arithmetic to an earlier point in the child's experience and the inclusion of new content in the mathematics program are significant trends in the modern curriculum. It appears also

**Hazel Lundberg *is Coordinator of Research and Teacher Education, Peabody Demonstration School, Nashville, Tennessee.***

[1] J. Fred Weaver. "Basic Considerations in the Improvement of Elementary School Mathematics Programs." *The Arithmetic Teacher* 7:269; October 1960.

that more time will be spent on elementary mathematics. But the single greatest trend is one of attitude. Experimenters are seeking not one best curriculum or one best method of teaching. Rather they are attempting to find various acceptable and effective means of reaching a common goal. Ben A. Sueltz, of New York State University College, in discussing the new attitude, says:

> The older pattern of "explain-practice-perform" is being replaced by a new spirit, a spirit of adventure, of speculation, thinking, discovery leading to understanding and self-projected learning. It is this same spirit of adventure that has established a favorable climate for experiment. . . . The contribution of many of these experiments may be more in the spirit of discovery than in the significance of the mathematics learned. But the spirit of discovery . . . constitutes the true essence of mathematics. . . .
>
> If we could admit that a child is essentially a curious person who likes to explore, we would then have the keynote to learning mathematics. He is, in fact, similar to the adult mathematician and creative worker in that he naturally explores with facts and ideas and establishes conclusions. This is an inductive process.[2]

In his explorations the child is likely to employ both intuitive and analytic thinking, and both should be encouraged. Although effective intuitive thought depends upon mastery of the subject, the child may get the "correct answer" to a problem with little knowledge of how the solution was reached. Once he has reached it, however, he may or may not be able to rationalize or prove it. If he cannot, a teacher must help him find the relationships which made his solution possible.

It is generally agreed that teachers of elementary mathematics can do much to aid pupils to "discover" mathematical ideas for themselves. Suggested discovery techniques are numerous. Jerome S. Bruner of Harvard University states that among the many approaches which will help pupils in discovery are:

> . . . use of the Socratic method, the devising of particularly apt computation problems that permit a student to find regularities, the act of stimulating the student to short cuts by which he discovers for himself certain interesting algorisms, even the projection of an attitude of interest, daring, and excitement.[3]

This discovery approach to learning is utilized to different degrees in all the new programs.[4] Robert B. Davis of Syracuse University reports that classes beginning as low as grade three, with a full range of I. Q.'s, have achieved notable success with this nonexpositional method.[5]

Increasingly more people are thinking, talking, and writing about changes in elementary mathematics. The National Council of Teachers of Mathematics reports receiving thousands of letters seeking guidance. Experimental groups are finding it necessary to make special arrangements to answer requests for information. College and university professors often are engaged as consultants and speakers. Yet questions remain:

1. When should formal instruction in arithmetic begin, and how long should it continue?

[2] Ben A. Sueltz. "A Time for Decision." *The Arithmetic Teacher* 8:274-80; October 1961.

[3] Jerome S. Bruner. "On Learning Mathematics." *The Mathematics Teacher* 53:610-11; December 1960.

[4] National Council of Teachers of Mathematics. *The Revolution in School Mathematics*. A Report Prepared by the Regional Orientation Conference in Mathematics. Washington, D. C.: National Council of Teachers of Mathematics, 1961. p. 79.

[5] Robert B. Davis. "The Syracuse University 'Madison Project.'" *The American Mathematical Monthly* 67:178-79; February 1960.

2. How can the child's thinking best be guided from the physical world of objects to the realm of mathematics?

3. To what extent are teaching systems such as the Stern and the Cuisenaire helpful in developing meaningful mathematics?

Answers to many of the questions are, at best, tentative. Experimental groups are providing information which will help to make sound judgments; but improvement of the curriculum, like the growth of mathematics itself, is continuous, and new improvements will replace former ones. Research demands the cooperation of mathematicians, teachers and psychologists, since most experimentation in arithmetic must take into account not only the mathematical value of the question being investigated but also principles of learning, methods of presenting materials, and age and ability levels of children. Henry Van Engen, University of Wisconsin, described the problem as being enormously difficult.[6]

The hypothesis that mathematics can be taught effectively in some form to any school child at any stage in his development [7] appears to be supported by findings of experimental studies. Data indicate that children in the elementary grades are able to handle abstract mathematics at a much earlier age than was thought possible a few years ago. Professor Davis reports that in the Madison Project remarkable achievement with abstract mathematical ideas has occurred in grades four and five.[8] Similar successes have been reported by David Page at the University of Illinois, by W. W. Sawyer at Wesleyan University, and by teachers in experimental schools using the materials of the School Mathematics Study Group (SMSG).

## Basic Concepts

As a basis for constructing an improved program, it is necessary to know what arithmetic knowledge the kindergarten child possesses. Studies designed to discover some of the specific number concepts possessed by children as they enter kindergarten were conducted in the public schools of Livonia, Michigan, and in the demonstration school of San Francisco State College. Two tests were used. The first was an individual oral interview requiring responses such as abstract counting by one and ten and rational counting by one and two. The second test was a picture-test requiring responses such as recognition of geometric figures and telling time to the full or half hour.

An examination of the data from these studies reveals very similar results. All children could count by one; the mean was approximately 19 for both rote and rational counting. Approximately 25 percent counted by ten and approximately 20 percent had some ability in counting by two. Ninety-three percent recognized two items when flashed, and 21 percent recognized eight, the maximum number flashed. The majority were able to recognize the geometric figures of a circle and a square.

A high degree of skill was evidenced in solving word problems that involved simple addition or subtraction facts. As a result of the study, Corwin Bjonerud of San Francisco State College recommends an inventory test for entering kindergarteners and a planned arithmetic program beginning the second semester of the year. A first grade program then

[6] Henry Van Engen. "The Status of Research in Arithmetic." From a Report Prepared by the Conference on Elementary School Mathematics, 1959.

[7] Jerome S. Bruner. *The Process of Education.* Cambridge: Harvard University Press, 1960. p. 33.

[8] Robert B. Davis, *op. cit.*, p. 178-79.

could be adjusted to care for variability, thus allowing for broader arithmetic experiences for many children.[9]

Inventories to determine the number knowledge of children as they enter kindergarten at Peabody Demonstration School [10] also show wide variation. Similar variance is found in grades one and two, indicating a need for a strong readiness program. In accordance with the hypothesis expressed by J. Houston Banks, George Peabody College for Teachers, that readiness is not altogether a function of maturation,[11] the program at the Demonstration School is designed to translate arithmetic ideas into the language and concepts of the child and then, through the relatedness of his knowledge, to lead him through successive stages of development.

## Usage and Structure

In the teaching situation at Peabody Demonstration School, no distinction is made as to whether arithmetic is developed from the standpoint of social usage or mathematical content. The children participate in varied activities: counting and distributing supplies, collecting lunch money, keeping records. In learning about measurements, children measure and often construct teaching materials such as clocks or scales. The child also experiments with certain concrete materials, which have the properties of abstract numbers. Each experiment illustrates a mathematical relationship, for example, "the 3 needs 4 to make 7 and the 5 needs 2." In this way the child discovers the number facts which he is expected to learn and, with help from the teacher, he relates them to daily living.

Both the social usage and the structural approaches, together with materials suitable to each, are used throughout the elementary grades. The Cuisenaire system is used in the teaching of fractions and is now being extended to the teaching of other concepts. Arithmetic is sequential in nature, and the abstract phase of the subject determines placement of content in the curriculum. Emphasis is upon understanding; learning is not considered complete until the mathematical relationships are established and the generalizations are projected to other situations.

In the spring of 1958, Newton Hawley and Patrick Suppes of Stanford University conducted a successful experiment designed to teach geometry material to primary children. There are now more than 5000 children under the supervision of the project, and hundreds of others are in schools which are conducting similar programs of their own. Professor Hawley states that the children who study geometry develop a genuine intellectual interest in the subject. This fact is important in itself. Since geometry is more typical of mathematics as a whole than is arithmetic, it is desirable that this branch of mathematics be introduced early.[12]

An experiment, patterned upon the program outlined by Professor Hawley, is being conducted with third grade children at the Demonstration School. Some of the features of the program are that it is designed for the entire class; it replaces

[9] Corwin E. Bjonerud. "Arithmetic Concepts Possessed by the Preschool Child." *The Arithmetic Teacher* 7:347-50; November 1960.

[10] Peabody Demonstration School is the laboratory school for George Peabody College for Teachers, Nashville, Tennessee.

[11] J. Houston Banks. *Learning and Teaching Arithmetic*. Boston: Allyn & Bacon, Inc., 1959. p. 10.

[12] Newton S. Hawley. "The Geometry Project." A mimeographed report, Stanford University, April 17, 1961.

none of the regular arithmetic; and it requires that the child read the instructions in order to carry out the steps in the constructions and in other exercises. The program has produced very satisfactory results.

Other experiments in elementary mathematics at Peabody Demonstration School have to do with the teaching of SMSG materials. The materials are developed from the viewpoint of structure; and beginning in grade four, the child is introduced to the language and notation of sets, some of the properties of real numbers, and topics from algebra and geometry. In grades five and six these topics are developed to greater depth, and new topics are introduced. By the time the child has finished the sixth grade, he will have studied the behavior of numbers, and he will know that mathematics deals with systems. The new mathematics is proving successful. The response of the children is excellent from the viewpoint of both interest and achievement. A noticeable result is the children's ability and willingness to attack new problems.

Similar success with the elementary SMSG materials has been experienced by Max Vann, Supervisor of Mathematics, Chattanooga, Tennessee, who taught selected units to approximately 5300 students in 176 classes. His television presentations were followed up by regular classroom teachers. Mr. Vann reports:

> The enthusiastic response to these programs on the part of both teachers and students far surpassed our expectations.
>
> Teachers report that far more students are interested in the new mathematics than were interested in the strictly traditional material. Teachers stated that while they themselves might have been hesitant about accepting the new material, the enthusiasm of the students served as a stimulus that caused them to begin studying the new concepts in earnest. Interest was further indicated by the fact that the teachers requested that a class in the newer mathematics be designed and taught for elementary teachers.[13]

The central theme in a program to improve school mathematics must be that whatever is taught shall be meaningful and significant to the learners.

[13] Letter from Max Vann, of Chattanooga, Tennessee, November 24, 1961.