Mathematics Education

BOTH in extent and depth, the past five-year period has been one of unprecedented study and investigation in mathematics education. The studies have involved mathematicians and teachers in all parts of the country, and they have been concerned with the teaching of mathematics at all levels from kindergarten to college.

An attempt is made in this brief review to select examples of research illustrating (a) methods of attack on four problems now of special concern, and (b) some of the results. The problems are:

1. What content and sequence will provide the best preparation for needs of tomorrow?
2. How do children learn mathematics?
3. What organization for administration of the mathematics program is most efficient?
4. How can in-service education be provided on a continuing basis?

Course Content

Studies of the mathematics curriculum for grades 9 through 12 were started at the University of Illinois soon after 1950, and several years later at the University of Maryland for grades 7 and 8. These projects were supported by the Carnegie Corporation of New York. Other investigations have been conducted at Stanford University, Southern Illinois University, Ball State Teachers College, and Syracuse University (now working with Webster College in St. Louis).

The major national effort in mathematics education is that of the School Mathematics Study Group (SMSG), supported by the National Science Foundation. A considerable part of the efforts of SMSG has been directed toward the preparation of sample courses (now available for grades 4 through 12 and in process of preparation for earlier grades). SMSG has also prepared extensive materials for teachers. The SMSG work has had the advantage of a team approach, involving hundreds of teachers and mathematicians in all parts of the country.

All of the projects in the development of new course materials for mathematics for grades K through 14 have provided for classroom try-out of these materials, for reports from teachers who have used the materials, and for modification of the materials based on teacher reports. This represents an important and new kind of research in mathematics education. While in most instances it is not controlled research and may not meet certain standards of educational research, nevertheless the practice of trying out new
materials in the classroom, with revision based on try-out experience, is a method
with implications for future curriculum development not only in mathematics, but in other disciplines as well.

In a number of instances there have been controlled studies of new materials. The hypothesis usually tested is that, on traditional mathematics tests and on tests covering the new mathematical topics, students who study the new materials do as well as students studying traditional materials. Such investigations have been conducted by the University of Maryland Mathematics Project for grades 7 and 8, and by the Minnesota National Testing Center for SMSG courses and some of the courses prepared at the University of Illinois.

Students using the new materials score well on tests covering these new topics, while students using the traditional materials are not successful on these tests.

On traditional tests, the differences are often not significant even for the lower group of students. Reports of these studies have not appeared in the literature, but copies can usually be obtained by writing to project sponsors.

A carefully prepared analysis of the new programs in mathematics has recently been published by the National Council of Teachers of Mathematics under the title, *The Revolution in School Mathematics* (1).

**How Children Learn**

Recent research at the University of Maryland has been concerned with how children learn mathematics. The instrument used in the research has been programmed instruction, since this instrument provides a way to control more satisfactorily teacher and other variables. An attempt has been made to find an-

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swers to such questions as the value of repetition and the importance of using integrating statements. Tests were made in controlled situations at the seventh grade level, on which some pupils had five exercises of a given type and other pupils only one exercise of this type. Integrating statements carefully point out the relationship of a new topic to topics previously studied.

A first examination of data collected appears to show that the best results were achieved when drill and integrating statements were combined; there was no significant difference in results involving increased drill alone or involving an increased amount of integration alone; and there was no significant difference in achievement of the students of lower ability and those of higher ability, suggesting that programmed instruction may bring about satisfactory achievement for students usually less successful in mathematics, due to the fact that in programmed instruction they must study every step and can do so at their own speed.

In other studies at Maryland, gifted students in the eighth grade were taught elementary concepts of convergence and divergence of series, and a similar group of students was introduced to mathematical induction. In the instruction on convergence and divergence of series, the teacher-taught class showed superior achievement, but when the researcher equated the additional amount of time used in the teacher-taught class with the time used in classes taught by programmed instruction, there were no significant differences in achievement. Complete reports of the Maryland studies will appear in the literature later this year.

At the University of Arizona several studies of programmed learning and teaching machines are under way. The studies are concerned with the teaching of ninth-grade algebra. Areas of investigation include techniques, concepts, reasoning and appreciation. The method involves variation in standard classroom procedures. Quite a number of those who have found enthusiasm for the use of programmed learning and/or teaching machines suggest that these methods of instruction might be used effectively, not as regular classroom procedure but in special instances and on special occasions for special topics. The Arizona studies are under the direction of Arthur Steinbrenner.

Promising studies in teacher education in mathematics have been reported by Easley at the University of Hawaii and Brown at the University of Delaware (2). At Hawaii, freshman prospective teachers start their orientation by teaching some of the new courses in mathematics...
to small groups of pupils. An investigation of common elements in teaching problem solving in mathematics and social studies is under way at Delaware. These studies also should contribute to our knowledge of how children learn mathematics.

An interesting study of the use of short cuts in computation has been reported in an unpublished thesis prepared at Ohio State University by Jacquelyn Byers. Miss Byers found that there are short cuts in computation which lend themselves to effective classroom presentation, and that these short cuts contributed to the understanding of such basic mathematical principles as place value, the commutative law, procedure for removing symbols of grouping, subtraction axiom, squaring the binomial, and the nature of proof.

### Studies of Administration

In 1956-58, the Science Teaching Improvement Program of the American Association for the Advancement of Science sponsored a study on the use of science and mathematics counselors (3). For purposes of the study, a counselor was defined to be a full-time staff member associated with a college or university science or mathematics department who was free to spend all of his time visiting high schools and working directly with teachers. Centers for the studies were located at the University of Nebraska, the University of Oregon, Pennsylvania State University, and the University of Texas.

Both in questionnaire answers and in interviews, teacher and administrator opinions were highly laudatory. The only major difficulty recorded was that there were not enough counselors. Interviews revealed that the greatest effects arose from improved morale and intellectual stimulation. Those responsible for the study consider the provision of science supervisors in state departments of education, under the National Defense Education Act, to have resulted in part from the success of the science counselors who participated in the two-year study.

A more recent study on the use of special teachers of science and mathematics in grades 5 and 6 was conducted by the Science Teaching Improvement Program, with the cooperation of the school systems of Cedar Rapids, Iowa; Lansing, Michigan; Washington, D. C.; and Woodford County, Kentucky. A preliminary report on the first year's activities has been made (4), and a final report will be published in the spring. The study showed that there was an advantage in the use of special teachers of science, but that there were no significant differences in achievement in mathematics. The achievement of one group of students with special teachers in science and mathematics was significantly greater in social studies, but not in science and mathematics. An explanation could be (this explanation was given by the teachers) that since the teachers did not have to prepare for science and mathematics, they had more time to make adequate preparation for social sciences. This year a pilot study involving schools in the area from Washington, D. C., to Wilmington, Delaware, in which the SMSG materials for grades 4, 5, and 6 are being used with special teachers, is under way. The pilot study also includes investigation of the use of team teachers and resource teachers.

Frank H. Anderson, mathematics supervisor of the Phoenix, Arizona, school system, reports (5) an experiment with large classes and student assistants. His conclusion is that there was no loss in
achievement for students in the large classes. The results seem to emphasize that success in this study was in considerable part due to the quality of the particular teachers involved.

Teacher Education

In addition to widespread investigations in elementary and secondary school teaching of mathematics, there is also universal interest in the improvement of teaching mathematics at the college level, especially in teacher education. In a recent issue of the *Review of Educational Research* (6), no less than 54 references appear in the bibliography on teacher education in mathematics for the three-year period of the report.

In a number of states, investigations are being made of teaching mathematics by television. These studies are in progress at the University of Nebraska, in Oklahoma under the sponsorship of the Oklahoma Frontiers of Science, and at the University of Wisconsin. The Wisconsin study combines the teaching of elementary mathematics classes with appropriate in-service work for teachers.

At Ohio State University a careful analysis was made of the effect of academic year institutes on 66 experienced mathematics teachers in four institutes. It was concluded that the teachers acquired a greatly increased interest in the study of mathematics and in further graduate work for advanced degrees; that, for the most part, the institute did not serve to attract teachers away from teaching at the high school level; and that what the teachers learned at the institutes, which was largely mathematics, brought about improvement in their own teaching. Their school colleagues at home reported that the participants were ready to contribute con-

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structively to the improvement of the mathematics program in the schools, but that they had little effect on other teachers.

The School Mathematics Study Group is sponsoring research on factors that contribute to teaching success in mathematics. Consideration is being given to such factors as the teacher's preparation in mathematics and his enthusiasm for mathematics as a subject. Reports on these studies will be made at a later date.

References


—JOHN R. MAYOR, Director of Education, American Association for the Advancement of Science, Washington, D.C.

Primary Arithmetic

(Continued from page 378)
produced as a table set was separated into equivalent sets. Eight children at a table were separated into four sets of two. This led to finding the attendance that day by counting by two's.

The concept of fractions had a beginning by finding that half the children in the group were boys and half were girls. They saw that Billy was one out of five children at his table, that Billy and Pete were two children out of five at their table and so on.

Estimation was invited by the question, “Knowing how many children are here when everyone is present, without counting, how many children would you estimate are here today?” ... “How many would you estimate are absent?”

The concept of proof ran through the attempts at solution of this idea-situation as children compared a variety of approaches and as the teacher questioned “why” to their process. The interrelationship of the concepts involved in the solution of the problem of attendance is obvious. The idea-situation provides the context for development of many concepts and insures a structure for relationships of concepts to take place.

Apart from problems arising from a group idea-situation, there are the child's individual idea-situations. He may need to figure out bus fare or make change. He may need to find out about units of measure as he works on a wood project, makes an aquarium, or plans refreshments for a party. Time must be given to help individuals work out their personal arithmetic problems. The teacher must become aware of problem possibilities in situations as he listens to children. He must teach by providing questions and materials to help the child solve his problems. Given this setting of group and individual idea-situations and a pattern to discover structure, children are free to develop broader and deeper understandings of basic arithmetic concepts.

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