# GPA The Pedagogic

• "Jerry Klotz is ineligible to play ball. His Grade Point Average is seven hundredths below the minimum."

• "I can't enter State this fall till I get that 2.23 up to 2.25."

• "If I don't get up to 2.00 this term I'm out."

• "The university to which I plan to transfer requires a 2.75 for entering graduate school."

STUDENT comments such as these cause one to wonder if we as educators apply mathematics in any such fashion as the mathematics we teach, or were taught. Clearly entrance requirements, athletic eligibility, and the very right to remain in school are preferably expressed in terms which are reasonably easy to compute, to understand, and to apply. Furthermore, it is helpful to have at some predesignated levels limiting criteria for purposes of selection, retention, and rejection. However, this writer contends that to arrive at a grade point average of, for instance, 2.54 as an output from a grade card input of rough whole numbers on the order of 4,3,2,1, and zero (A, B, C, D, and F) is mathematically indefensible and in many cases leads to gross injustice.

Mathematics people refer to the reliability of figures, distinguishing between precision and accuracy as two separate factors to consider in measuring reliability. Precision is a function of the number of decimal places to which, for instance, a measuring device is really capable of recording whatever unit of measure is employed.

This is to say that 6.573 is a more *precise* measurement than is 6.57 inches.

Accuracy, on the other hand, is a function of the number of significant figures, or numbers known to be reliable. Thus, 0.0004 contains but one significant figure and is therefore less *accurate*, considering its relation to the whole, than is the number 0.52 even though 0.0004 is more *precise* than 0.52. A change of one number, i.e., 0.52 to 0.53, is a much smaller percentage (2 percent) of error than should 0.0004 be judged to read 0.005 (25 percent) on a measuring device. The "two" in the number 0.52 can therefore be regarded with greater confidence than can the four in 0.0004.

## A Fallible Process

A conscientious teacher making a grade is assigning a literal value (convertible to a numerical value) to something that has been measured—achievement, progress, or whatever. His measuring devices usually consist of tests. Human variables being what they are, the tests on occasion being no more valid than they are, the size of many classes precluding desirable evaluation procedures, and worst of all because of the necessity to observe ironclad tradition, his instructions are merely to assign letter grades on the order of A, B, C, D, F. These correspond to whole numbers per credit of 4, 3, 2, 1, 0.

The teacher's measuring device is read to a precision of one whole number, no

# Numbers Game

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decimal places, and to an accuracy of one significant figure. The GPA computers blithely convert these measurements into figures carried to a precision of two decimal places and to an accuracy of three significant figures, and in some cases to three decimal places and four significant figures! Upon these last few mathematically unsupportable hundredths have rested Go—No Go decisions which recently could take a student from the classroom to the rice paddies of Viet Nam and which presently are worshipfully observed in making some rather highly important judgments.

A question faced by those who perform measurements and computations is that of how much of a computed result can honestly be retained as being reliable. Were we to measure the length of a surface with a tape marked in whole inches only and the width with a tape graduated to hundredths of an inch, could we say then that a length of 141 inches times a width of 5.26 inches yields an area figure of 741.66 square inches? The mathematician would tell us that in multiplication and division we retain a product with no more significant figures (ones known to be reliable) than the least *accurate* of the figures used in obtaining the product or quotient. There are three significant figures in each of the above input numbers, so an answer of 742 square inches is all we can claim in computing the area.

Should a third dimension of .01247 inches be added and we were to perform the operation $141 \times 5.26 \times .01247$, we could retain a volume measurement of only three significant figures or 9.25 cubic inches, not 9.2485002 cubic inches. Quotients are reckoned the same way as products. The result that can be retained is contingent upon the less *accurate* or the number with the fewer significant figures posed as either the dividend or the divisor. (Sums and differences retain an answer in keeping with the least *precise* of the individual numbers; 0.07563 plus 0.061 equals 0.137.)

According to the foregoing, it is erroneous to compute a grade point average as shown:

| Course | Grade | Credits | Grade Points |
|---|---|---|---|
| Mathematics | A | 3 | 12 |
| English | C | 3 | 6 |
| Sociology | B | 3 | 9 |
| Physics | C | 4 | 8 |
| Health | B | 2 | 6 |
| | | 15 | 41 |

$$\frac{41}{15} = 2.73 \text{ GPA}$$

The dividend and the divisor each are represented by two significant figures. Therefore a GPA of 2.7 is the maximum number of decimal places to which this rating should

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ever be applied in making important decisions.

Furthermore, to describe a human being as one with a GPA of 2.73, for example, is to strike a rather improbable average in many instances. A single figure is drawn from grade data derived from a large number of activities, oftentimes requiring completely unrelated competencies. Grades in English Composition and in Mechanics of Fluids, grades in Spanish and in College Physics, grades in History of Western Civilization and in P.E. 190 Golf—all these combine to say, "This person is a 2.73 person." He is less than a 2.74 person and superior to a 2.72 person. A plausible analogy would be to take the prices of prime rib, pork chops, ground beef, lamb chops, and liverwurst, average these prices, and solemnly quote the price of meat at 89.46 cents a pound.

Most teachers are frank in acknowledging that they can never be sure the grades they turn in are fair to each individual. To issue grades of A, B, C, D, or F is a trying assessment at best. Nevertheless, many students are crushed by a system that does not distinguish between a marginally high letter grade and a marginally low letter grade. A "B" is a "B" whether it narrowly missed being an "A" or whether it hovered in the gray zone between "B" and "C" prior to emerging as a "B." In the "C" grades the inequities are likely to be more numerous, the range from near "D" to near "B" being greater.

Students accosted by the writer indicated they were both cognizant and concerned about this. A few merit points in a single course can have unwarranted effect upon the overall grade point average of an individual student. The end result is reminiscent of our electoral college system of counting votes. Certainly a worthwhile project would be to strive for an improved way to express grades—and votes!

## Inferences

What inferences or conclusions might logically be drawn? First, *grade point averages should not be computed beyond the first decimal place, or in other words, beyond tenths*. This contention rests upon mathematical principles clearly expressed in texts devoting space to the subject. Greater accuracy is not achieved by dividing measured values to increasing numbers of decimal places. To do so is a procedure not worthy of trained educators.

A second inference is presented which is much more likely to be controversial and which is more difficult and perhaps impractical to apply. *Grades in individual courses should be expressed in two significant numerical figures*, letter grades being abolished. Thus a student with a high "C" in a given course would earn a higher GPA for a given term than would his counterpart with a low "C" in the same course, all other grades being identical.

| Course | Grade | Credits | Conventional Grade Points | Highest Possible | Lowest Possible |
|---|---|---|---|---|---|
| Mathematics | A | 3 | 12 | 13.5 | 10.8 |
| English | C | 3 | 6 | 7.5 | 4.8 |
| Sociology | B | 3 | 9 | 11.5 | 7.8 |
| Physics | C | 4 | 8 | 10.0 | 6.4 |
| Health | B | 2 | 6 | 7.0 | 5.2 |
| | | | 41 | 49.5 | 35.0 |

Conventional GPA, 41/15 equals 2.7
Highest Possible GPA 49.5/15 equals 3.3
Lowest Possible GPA 35/15 equals 2.3

Table 1. Range of Grade Point Averages

For sake of discussion suppose that letter grades were equated with numerical ranges as follows:

| | | | |
|---|---|---|---|
| A | 3.6 | to | 4.5 |
| B | 2.6 | to | 3.5 |
| C | 1.6 | to | 2.5 |
| D | 0.6 | to | 1.5 |
| F | −0.5 | to | +0.5 |

The range of grade point averages for the instance cited earlier is shown in Table 1.

By this illustration it can be seen that students all rated the same with a 2.7 average would instead find themselves rated from 2.3 to 3.3. Would this not be a more equitable system? A random grade card might appear as shown:

| Course | Credit | Grade | Grade Points |
|---|---|---|---|
| Mathematics | 3 | 3.9 | 11.7 |
| English | 3 | 1.7 | 5.1 |
| Sociology | 3 | 3.1 | 9.3 |
| Physics | 4 | 2.4 | 9.6 |
| Health | 2 | 3.5 | 7.0 |
| | 15 | | 42.7 |

$$\frac{42.7}{15} = 2.8 \text{ GPA}$$

The question arises, "If a teacher can never be certain in assigning appraisals designated by merely five letter-grades, how can he distinguish student differences to the nearest tenth? Obviously he cannot be certain of each case. However, should all the "B" students, for instance, be listed in rank order, the extremes would in all likelihood be discernible and justice would result more frequently than under present letter-grade systems. After alignment in rank order, the students could be assigned numerical grades to the nearest tenth based on this rank order within a given numerical range.

To present a challenge to registrars sufficient to elicit cries of anguish, grade point averages would be more meaningful if presented several in number, covering logical clusters of subjects. One GPA might be in engineering subjects only, another in humanities or linguistic courses taken, and so forth, such that a more relevant composite profile would be expressed than to describe one as a 2.7 person.

Whether the foregoing propositions have merit or not is probably academic in the face of the ever-present squelch: "We don't have anyone who can feed this kind of information to the computer." This, incidentally, is more forthright than the occasional comment, placing the blame whence there will arise no vocal objection, "The computer can't handle it that way." Seemingly at times we are being guided by our tools and by tradition to an unfortunate degree.

In conclusion, much excellent thinking has been done regarding grades, grading, and grade point averages. One result is that the "Let's Do Away With Grades" group is becoming increasingly vocal. This could be a solution, providing an alternate scheme can be developed to indicate either achievement or potential in readily understandable form.

After all, the students we teach are all different from one another. The prospective employer and the senior institution are interested in knowing how a particular student compares with his fellows. A fallible system is certainly better than no system whatever. As we improve instructional techniques, and as we improve our testing and measurement procedures, perhaps we can also improve the way we record the results we achieve with our students. □