

# Science and Sense in School— Mathematics Skill Instruction



Leroy G. Callahan

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*Rather than swinging from one extreme to another, schools should use good sense and scientific research to decide what, when, and how mathematics should be taught.*

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Because of its dual nature, arithmetic, perhaps more than any other school subject, is affected by the swirl of social pressures. Arithmetic can be viewed as a highly organized system of abstract, related ideas—a discipline with a rich cultural heritage worthy of study for its own sake. It can also be viewed strictly from a functional utilitarian perspective—as a tool subject.

School instructional procedures are often affected by the point of view. Direct, no frills procedures with heavy doses of drill and practice are associated with the functional perspective; indirect finesse marks attempts to have students learn arithmetic as a system of abstract related ideas. Society's educators "center" their focus toward one of these aspects of arithmetic at the

expense of the other interrelated aspect. Consequently, periodic changes from one point of view to the other tend to sap energies needed to improve mathematics instruction.

These swings often give observers of the school mathematics scene a feeling of *déjà vu*. In reaction to the abstractness associated with the modern math of the sixties, read today's publishers' brochures promoting "real life" applications and the appeal of realistic photographs used in their current textbooks. Then consider the following observation of Charles H. Judd over a half-century ago:

As I read the modern textbooks. . . . There is concreteness on every hand. There are pictures of people buying and selling, there are reproductions of checks and bills of lading, there are problems from

geography and baseball, but the fact that the number series has a character and regularity of its own is pushed as far into the background as the pages will permit. The new textbooks might have on their covers a picture of a tool as symbolic of their content. They seem to announce in the loudest tones the beliefs that all the good things in modern civilization are real, tangible objects; number is merely something to be used now and then as a vague and unsatisfactory substitute for things that are concrete and substantial and truly important.<sup>1</sup>

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I believe we know better than to allow the wild shifts in view on what, when, and how mathematics instruction is practiced in our schools. The following comments on those matters must be broad and general. However, the simple essence is a call for "decentration" in views and a search for a dynamic equilibrium in educational perspective that will generate a balanced progress in improvement of instruction in school mathematics.

### What Mathematics Skills Should Be Taught?

What are the basic skills in mathematics? Both the National Council of Teachers of Mathematics and the National Council of Supervisors of Mathematics have recently presented position statements on basic skills.<sup>2</sup> Their statements reflect a concern for the inappropriate tendency to look "back" for what was basic, and they attempt to discern the essential mathematical needs of adults in the present and future. They also reflect a concern with the tendency to equate essential mathematical skills with computational skills. The statements promote basic skill instruction within the broader context of the total mathematics program. The scope of essential mathematics skills ranges from "Problem Solving" to "Computer Literacy"; from "Appropriate Computational Skills" to

"Using Mathematics to Predict." Ten skill areas are presented, all basic to pupils' development of ability to reason effectively in varied situations. School instruction would be concerned with developing proficiency, applying skills in significant situations, and presenting skills in a context that provides students with opportunities for glimpses of mathematics as an organized system of related ideas. With this scope and substance of skills, what about instructional practices?

"Instruction is, after all, an effort to assist or to shape growth."<sup>3</sup> A well formulated, validated theory of mathematical instruction does not exist. That fact makes discussion of "when" and "how" instruction will most assist or shape student growth a somewhat speculative, not scientific, undertaking. But the practice of teaching mathematics to students has a long history, even though the formal study of its teaching practices is quite recent. Though there is a lack of a concise science of teaching, that does not mean that experience has not provided some sense of teaching. Many instructional practices may be right because they make sense and may ultimately receive scientific validation. To use a homely analogy, chicken soup has been used for years as a remedy for many ills—the practice resting on beliefs, traditions, and pragmatic results. Only recently has some scientific validity been found for the practice. Following are some "chicken soup" observations regarding "when" and "how" decisions in mathematics instruction. They present a perspective for instructional practices to assist or shape mathematical growth of students—though they may lack scientific verification.

### When Should Mathematics Skills Be Taught?

When does instruction have the greatest effect on a student's growth in mathematics skills? The sensible response is simple; it is when the

<sup>1</sup> Charles H. Judd. "The Fallacy of Treating School Subjects as 'Tool Subjects.'" *Selected Topics in the Teaching of Mathematics*. Washington, D.C.: National Council of Teachers of Mathematics, Third Yearbook, 1928. pp. 1-10.

<sup>2</sup> See: "Position Statement on Basic Skills." *Arithmetic Teacher* 25: 18-22; October 1977.

<sup>3</sup> Jerome S. Bruner. *Toward A Theory of Instruction*. New York: W. W. Norton, 1968. p. 1.

student is ready for instruction, and the instructional practice is appropriate for the student.

First consider student readiness. Over 50 years ago, Whitehead wrote, "The principle is merely this—that different subjects and modes of study should be undertaken by pupils at fitting times when they have reached the proper stage of mental development."<sup>4</sup> For many years, science has been attempting to impose precision on the principle. All too often poor science has led to poor practice, or the sensible principle has been distorted to nonsensical practice. The Committee of Seven studies directed by Washburne<sup>5</sup> attempted to determine the stage of mental growth at which students could most effectively learn, for example, subtraction facts. Though the experimental procedures were questioned,<sup>6</sup> the implications from their results—that large numbers of students were not "ready" for learning mathematics topics—were often imposed without question. The practical effect was "pushing back" the introduction of many mathematics topics. This type of overzealous application of "readiness" was probably instrumental in having the Council for Basic Education list the "dogma of readiness" as one of the "Seven Deadly Dogmas of Elementary Education."<sup>7</sup>

More recently there has been a tendency to take scientific analysis of mathematical content and processes in the form of taxonomies and hierarchies, combine them with the readiness principle, and produce "systems" of instruction. These systems generally incorporate vast lists of behavioral objectives, with large numbers of associated diagnostic tests and sequenced, student-paced, instructional modules. Instructional "systems" have gained a degree of popularity in spite of some negative evidence regarding their effect on assisting, or shaping student growth in mathematics.<sup>8</sup> Again, overzealous applications of scientific rigor to "systems" development may lead to *rigor mortis* in student development. Whitehead observed the problem in his own time. He wrote, "The uncritical application of the principle of the necessary antecedence of some subjects to others has, in the hands of dull people with a turn for organization, produced in education the dryness of the Sahara."<sup>9</sup>

So what about readiness for instruction? It is still a sensible concern for classroom teachers of

mathematics. In its application, on the right hand, we must develop a patience with science in accumulating knowledge that may bring precision to the principle for instructional practice. In the meantime, we proceed on the left hand to apply the principle in sensible ways. Consider the following situation. To determine whether an intermediate grade student is "ready" for effective instruction in a mathematics skill area, a series of

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diagnostic assessments are administered to see if antecedent knowledge is in place. Suppose the skill was division of mixed numbers by a common fraction, such as  $1\frac{1}{2} \div \frac{1}{4}$ . Suppose further that the assessment data indicated little or no formal knowledge of numbers in the form  $1\frac{1}{2}$ ,  $\frac{1}{4}$ , and little knowledge of the division operation. Accordingly the decision is made not to introduce the topic but to go back to the antecedents.

But teaching sense tells us that knowledge about such topics is not all or nothing, have or

<sup>4</sup> Alfred North Whitehead. *The Aims of Education*. New York: The New American Library, 1949. p. 26.

<sup>5</sup> Carleton W. Washburne. "The Grade Placement of Arithmetic Topics." Twenty-ninth Yearbook, National Society for the Study of Education. Bloomington, Illinois: Public School Publishing Co., 1930. pp. 641-70.

<sup>6</sup> William A. Brownell. "A Critique of the Committee of Seven's Investigations on the Grade Placement of Arithmetic Topics." *Elementary School Journal* 38: 495-508; March 1938.

<sup>7</sup> See: *A Decade of Comment on Education*. Washington, D.C.: Council for Basic Education, 1966. pp. 6-12.

<sup>8</sup> Harold L. Shoen. "Self-paced Mathematics Instruction: How Effective Has it Been?" *Arithmetic Teacher* 23: 90-96; February 1976.

<sup>9</sup> Whitehead, *op. cit.*, p. 28.

have not. Most students by the intermediate grades know something about  $1\frac{1}{2}$ ,  $\frac{1}{4}$ , and the operation of division. Although unable to respond meaningfully to the symbolically presented task like  $1\frac{1}{2} \div \frac{1}{4} = N$ , many students may be able to respond with meaning to a sequence of teacher questions like:

- How many quarters are in a half dollar?
- How many quarters are in one dollar?
- How many quarters are in one dollar and one-half?
- How many quarters are in two dollars?

From the soundings for structural bedrock, the skilled teacher can help the student make a written record of the knowledge. It may be a shorthand like:

- $\frac{1}{2} \rightarrow 2$
- $1 \rightarrow 4$
- $1\frac{1}{2} \rightarrow 6$
- $2 \rightarrow 8$

Such a written record presents a compelling suggestion about pattern and order in arithmetic. It could provoke questions requiring extrapolating or interpolating from the record. Students may glimpse the power of mathematics from the simple arithmetic activity. In reverting to the skill development aspect, the student could then be introduced to the conventional notation for his knowledge:

- $\frac{1}{2} \div \frac{1}{4} = 2$
- $1 \div \frac{1}{4} = 4$
- $1\frac{1}{2} \div \frac{1}{4} = 6$
- $2 \div \frac{1}{4} = 8$

The framed notations might well offer the beginning for understanding the convenience of using the reciprocal algorithm for division with fractions.

The point to be made is that it is not sensible to be exclusively preoccupied with necessary content antecedents when deciding on when mathematics skills can be taught. Such preoccupation can deprive students of the opportunity for much valuable learning. The tendency to look back in determining "readiness" must be tempered by a confidence in appropriate instruction to compel students forward in learning. Vygotsky observed:

Therefore the only good kind of instruction is that which marches ahead of development and leads it; it must be aimed not so much at the ripe as at the ripening functions. It remains necessary to determine the lowest threshold at which instruction in, say, arithmetic may begin since a certain minimal ripeness of function is required. But we must consider the upper threshold as well; instruction must be oriented toward the future, not the past.<sup>10</sup>

But our sense of teaching should suggest a tempering of any exclusive use, or extreme dominance in use, of the concrete and informal procedures in mathematics skill instruction. Although the normative procedures across a broad cross-section of mathematics instruction may be dominated by the informal concrete activities, a vertical linear view of a specific topic strand should reflect a concern for continuous student growth from concrete toward abstract, from real things toward symbol representations, from informal to formal.

### How Should Mathematics Be Taught?

How instruction of a topic is undertaken may well be a more important practical consideration for teachers than precision in "when" a topic is taught.

Appropriate instructional techniques are important in assisting or shaping growth in mathematical skills. The scientific work of Piaget in describing intellectual stages of development has implications for instructional practices. At the elementary and early middle school years, the majority of students are between the pre-operational stage of development and the stage of abstract thought. Piaget refers to this span of years as the "concrete operations" stage. The implications are that at this stage learning should be active, and the activity should be with or on concrete models. He writes:

... The procedure that would seem indispensable would be to take as the starting point the qualitative concrete levels: in other words, the representation or models used should correspond to the natural logic of the levels of the pupils in question, and formalization should be kept for a later moment. . . .<sup>11</sup>

<sup>10</sup> Lev Semenovich Vygotsky. *Thought and Language*. Cambridge, Massachusetts: M.I.T. Press, 1962. p. 104.

<sup>11</sup> Jean Piaget. "Comments on Mathematical Education." In: *Developments in Mathematics Education*. Proceedings of the Second International Congress on Mathematics Education. London: Cambridge University Press, 1973. p. 85.

Whitehead described what he sensed as the rhythmic nature of the educational experience.<sup>12</sup> The experiences moved from the initial stage of romance to a stage of precision to a stage of generalization that is both a culmination of one cycle and commencement of a new learning. A student's knowledge of  $4 \times 4 = 16$  at the abstract symbolic level is the conclusion of one cycle that may have begun with some "romantic" explorations with arrays of blocks, but it can also be the commencement of a new cycle of learning about numbers that are perfect squares.

### From "Romance" to "Precision"

Indiscriminant or exclusive use of informal concrete instructional practices during the elementary school years may cause some dissonance in the rhythmic cycles of student growth in mathematical skills. A sixth-grade student may be excited about undertaking an informal exploration using numbers as they apply to a particular area of interest, but he/she quickly becomes discouraged and frustrated if only concrete informal

means mark the development of the arithmetic processes needed in the exploration. It is difficult to sell mathematics as one of the greatest labor saving schemes devised by humankind to students who must use their fingers or count tally marks on a piece of paper in their arithmetic applications. Within topical strands, there is a time for "romance," but there comes a time for "precision."

### Summary

So classroom teachers of mathematics practice within an interrelated field of stress set up by polar attractions along the "what," "when," and "how" dimensions of school mathematics instruction. In making determinations regarding "what" to teach, there are attractions toward the values of functional usefulness and logical structure of mathematics. In making determinations "when" instruction may most benefit growth, there are attractions toward the power of antecedent conditions of students and the power of appropriate instruction procedures. In making determinations

<sup>12</sup> Whitehead, *op. cit.*, p. 28.

## Math Supervisors Define Ten Basic Math Skills

Following are the ten mathematical skills that the National Council of Supervisors of Mathematics considers basic:

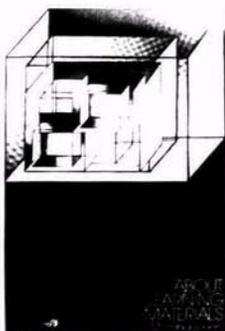
- **Problem solving**—Learning to solve problems is the principal reason for studying mathematics.
- **Applying mathematics to everyday situations**—Students should be encouraged to take everyday situations, translate them into mathematical expressions, solve the mathematics, and interpret the results in light of the initial situation.
- **Alertness to the reasonableness of results**—Students should learn to inspect all results and to check for reasonableness in terms of the original problem.
- **Estimation and approximation**—Students should be able to carry out rapid approximate calculations by first rounding off numbers. They should acquire some simple techniques for estimating quantity, length, distance, weight, and so on.
- **Appropriate computational skills**—Students should be able to perform addition,

subtraction, multiplication, and division with whole numbers and decimals. They should also be able to work with fractions and percents.

- **Geometry**—Students should learn the geometric concepts they will need to function effectively in the three-dimensional world.
- **Measurement**—Students should be able to measure distance, weight, time, capacity, temperature, and angles calculations of simple areas in metric and customary systems.
- **Reading, interpreting, and constructing tables, charts, and graphs**—Students should be able to read and draw conclusions from simple tables, maps, charts, and graphs.
- **Using mathematics to predict**—Students should learn how elementary notions or probability are used to determine the likelihood of future events.
- **Computer literacy**—Students should be aware of the many uses of computers in society.

\*The NCSM is an affiliated group of the National Council of Teachers of Mathematics.

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on "how" instruction is beneficial to growth, there are attractions toward the broad stages of student intellectual development and the rhythmic cycles of specific curriculum experiences. These stress fields are further affected by the press and pressures of the society the school serves. The changing economic, demographic, and political conditions in society generate changing forces that affect the press and pressure on professional decisions.

If progress in student mathematics learning is to be made, there is need to establish a stability in stance on school mathematics programs. Stability can come from "decentering" thinking from polar views to a point of dynamic equilibrium between the extremes. From experimentation and experience there has been some accumulation of knowledge regarding what is good and right in school mathematics instruction. The press of various forces often deters us from applying this knowledge in practice. But this is an age-old problem and goes well beyond school mathematics instruction. Perhaps no one has put it as well as Shakespeare's Portia in *The Merchant of Venice* when she said:

If to do were as easy as to know what were good to do, chapels had been churches, and poor men's cottages princes' palaces . . . I can easier teach twenty what were good to be done, than one of the twenty to follow mine own teaching.<sup>13</sup>

Science and sense give us the leads in knowing what is good to do in school mathematics; strong educational leadership and well prepared and concerned teachers are needed to do it. <sup>14</sup>

<sup>13</sup> From: William Shakespeare. *The Merchant of Venice*. First published in 1600 by James Roberts.



*Erroy G. Callahan is Professor, Department of Elementary and Remedial Education, State University of New York at Buffalo, Amherst, New York.*

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