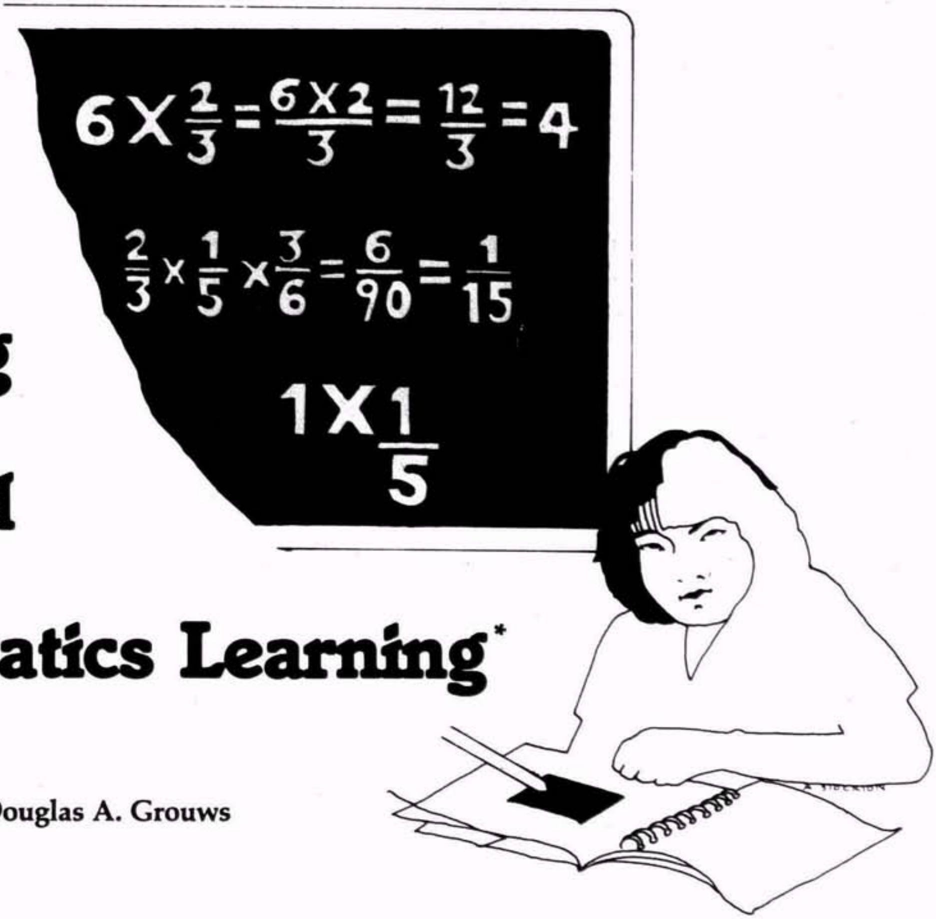

Teaching and Mathematics Learning*

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Student learning in elementary mathematics can be increased by some key teacher behaviors identified by research.

After conducting research on mathematics instruction in the elementary schools for the past several years, we now feel that extant data illustrate that the type of instruction teachers use can make an important difference in student learning. And these learning gains do not come at the expense of student interest in mathematics. It appears possible to change teacher behavior and to increase student learning, at least on some dimensions, without elaborate or expensive training.

Here we will summarize our past efforts and discuss needed research in this area. Reviews of the literature on mathematics learning and instruction can be found elsewhere (Fey, 1969; Riedesel and Burns, 1973; Dessart and Frandsen, 1973; and Cooney, in press).

Our initial research effort in this area began with a sample of over 100 fourth-grade teachers who taught in a middle-class urban school district and used the same textbooks. Pre- and post-test data on a standarized achievement test (fall and spring testing) were available for three consecutive years and thus a residual mean gain score could be computed for each teacher, and his or her level of effectiveness over time could be compared with other teachers.

To distinguish or to make comparisons among

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teachers, it is necessary to accept an operational definition of effectiveness. In our research we have used student test performance on a standardized achievement test in mathematics to estimate instructional effects. We know that a standardized score is not a perfect measure of classroom learning (the standardized test does not overlap completely with the content that has been taught) and that because teachers emphasize different content areas somewhat different results are produced for the same class with different achievement tests (see, for example, Porter and others, 1978).

Despite the fact that standardized tests have many difficulties, we felt (and still do) that there is

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reasonable consensus among teachers and standardized test developers as to what constitutes the mathematics curriculum. Hence, it is possible to use such a test to compare the impact of different instructional conditions upon student learning. Given that we have used standardized achievement tests to define effectiveness, information that we will present about how more "effective" teachers behave in the classroom is of course restricted to effectiveness in producing gains on these tests and to the more basic types of content that are assessed by such tests.

To estimate teachers' effectiveness we computed a residual mean gain score for each teacher each year using his or her students' pre- and post-test scores on the standardized test. We found that teachers varied considerably in their impact on students' learning, despite the fact that they were using the same textbooks and in many cases were teaching comparable students (that is, the mean class achievement scores were very similar in the fall). Our initial data set was thus a powerful demonstration of a teacher effect. Some teachers produced much more mathematics learning than did other teachers.

Teacher Performance Stability

It is important to note, however, that the stability of teacher performance as estimated by student scores on a standardized achievement test was low across years. For example, some teachers who were relatively high one year fell to the middle of the distribution the following year on the basis of residual mean scores (discussion of stability issues can be found elsewhere: Brophy, 1973; Good and Grouws, 1975a; Emmer, Evertson, and Brophy, 1979).

Some teachers fluctuate from year to year. It may be, as Good (1979) argues, that for these teachers subtle context variables (such as the particular students assigned to a teacher) or personal circumstances (some years one can devote more time to teaching than is possible in other years) may moderate the amount of student learning that is produced. However, there were some teachers who were stable and relatively high or low over several consecutive years.

At this point it would have been interesting to explore the reasons for the lack of stability in generating student learning. That is, we could have studied teachers who fluctuated from year to year over time to see if they *behaved* the same way in the classroom over consecutive years. However, our focus was placed upon observing stable teachers because we had an interest in identifying mathematics teaching strategies that were successful in the classroom. It was our assumption that those teachers who had a stable and relatively high or low level of effectiveness would be an excellent starting point for estimating the relative effectiveness of different teaching behaviors.

It is beyond the scope of the present paper to describe research samples and findings in detail. More detailed information can be found elsewhere (Good and Grouws, 1975b; 1977; 1979; and Ebmeier and Good, 1979). We did find that stable high and low teachers differed in their classroom behavior. That is, more and less effective teachers taught in different ways, and some of these differences in teaching behavior were consistent across the two groups of teachers (for instance, most of the effective teachers were found to perform a given behavior more or less often than the relatively ineffective teachers).

The naturalistic findings illustrated that more effective teachers, in contrast to less effective teachers (1) taught the class basically as a whole (a few students might be assigned individual work, but essentially the teacher had one instructional group); (2) presented information more actively and clearly; (3) were task-focused (most of the period was spent on mathematics, not socialization, and so on); (4) were basically nonevaluative and created a relatively relaxed learning environment (comparatively *little* praise or criticism); (5) expressed higher achievement expectations (more homework, faster pace, more alert environment); and (6) had fewer behavioral problems.

The observational work in the naturalistic study thus illustrated that it was possible to differentiate between the two groups of teachers and enhanced our interest in attempting to increase student performance. Next we wanted to see if it would be possible to build a training program based upon these effective teaching behaviors which might improve student mathematics learning.

The Need for Experimental Work

Although we were pleased with the naturalistic findings in the sense that they provided some clear contrasts between relatively high and low gain classrooms, we were aware of the fact that these were only correlational results and that they did not substantiate the fact that these teacher behaviors caused student achievement. It could be that behaviors not studied in our observational research were more directly related to achievement (more effective teachers plan more thoroughly and because of this they are more task focused, assign more homework, for instance).

We now wanted to see if more direct linkages could be established between the behaviors that had been identified in our observational naturalistic study and student achievement. Because of the expense involved in field testing a program, we wanted it to be as comprehensive as possible. Thus, in building the training program we integrated our results with those available from other process-product studies and those from previous experimental mathematics studies. The effort resulted in a 45-page training manual (Good and others, 1977).

The program, as pointed out elsewhere (Good and Grouws, 1979), is a *system* of instruction: (1) Instructional activity is initiated and reviewed in the context of meaning; (2) Students are prepared for each lesson stage to enhance involvement and to minimize errors; (3) The principles of distributed and successful practice are built into the program; (4) Active teaching is demanded, especially in the developmental portion of the lesson (when the teacher explains the concept that is being studied, its importance, and so on). An overview of the program is shown in Figure 1.

To test the program, we utilized 40 classrooms in the Tulsa, Oklahoma, Public School System. Roughly half of these classrooms were assigned to experimental and control conditions. Experimental teachers read the manual and were given approximately two and one half hours of training.

Figure 1. Summary of Key Instructional Behaviors

Daily Review (First eight minutes except Mondays)
- (a) Review the concepts and skills associated with the homework
- (b) Collect and deal with homework assignments
- (c) Ask several mental computation exercises

Development (About 20 minutes)
- (a) Briefly focus on prerequisite skills and concepts
- (b) Focus on meaning and promoting student understanding by using lively explanations, demonstrations, process explanations, illustrations, and so on
- (c) Assess student comprehension
  - (1) Using process/product questions (active interaction)
  - (2) Using controlled practice
- (d) Repeat and elaborate on the meaning portion as necessary

Seatwork (About 15 minutes)
- (a) Provide uninterrupted successful practice
- (b) Momentum—keep the ball rolling—get everyone involved, then sustain involvement
- (c) Alerting—let students know their work will be checked at end of period
- (d) Accountability—check the students' work

Homework Assignment
- (a) Assign on a regular basis at the end of each math class except Fridays
- (b) Should involve about 15 minutes of work to be done at home
- (c) Should include one or two review problems

Special Reviews
- (a) Weekly review/maintenance
  - (1) Conduct during the first 20 minutes each Monday
  - (2) Focus on skills and concepts covered during the previous week
- (b) Monthly review/maintenance
  - (1) Conduct every fourth Monday
  - (2) Focus on skills and concepts covered since the last monthly review

Experimental Findings

On the basis of observers' records it was found that the experimental teachers implemented the program very well (with the exception of certain recommendations concerning how to handle the development portion of the lesson). Given that experimental teachers did use the program, it was possible to "determine the effects of the program."

Pre- and post-testing with a standardized achievement test indicated that after two and one half months of the program the performance of students in experimental classrooms was considerably better than those in control classrooms. It was also found that the experimental students' performance increase continued at least for some time following the treatment. Regular end-of-year testing by the Tulsa Public School System (with a standardized test) indicated that approximately three months after the program had ended, the experimental students were still performing better than were the control students. Such findings are encouraging.

In addition to the standardized testing, we constructed a content test that more closely matched the material that teachers were presenting than did the standardized test. The results on this test also showed an advantage for experimental classes, although the differences between control and experimental classrooms were not as large as they were on the standardized test. Also, pre- and post-testing on a 10 item attitude scale revealed that experimental students reported significantly more favorable attitudes at the end of the experiment. Thus, the achievement gains did not come at the expense of student *interest* in mathematics. Finally, it should be noted that feedback from the teachers indicated that they felt that the program was beneficial and that they planned to continue using it.

Needed Research

The results indicate to us that it is possible to improve student performance in important ways in inner-city schools. Despite these encouraging findings, we do not want to imply that the methods used in our program are the most desirable way to teach mathematics. It would be worthwhile and important to investigate the effects of the instructional model in a wide variety of situations using additional outcome measures. For example, program effects on logical thinking, physical applications, consumer decision making, and problem solving would be a few of the many outcomes of interest. We currently have research in progress that focuses on verbal problem solving both as an outcome and as part of an instructional program. Whether or not the model would have desirable impact on outcomes not measured in our research is undetermined.

One important situation that we have not actively explored is the use of the program with small group instruction. Some of the teachers in the control condition who taught mathematics to groups of students achieved very good results. Indeed, one very important research question that *needs* to be answered at the elementary school level is how teachers who obtain good results using small group instruction behave in the classroom. We know from data we have collected that some teachers who teach small groups achieve better results than other teachers, but very little specific information is available on how these teachers behave. Information about *effective* small group instruction would seem to be a very important next step in trying to understand mathematics learning in the elementary school.

Additional refined research on the development portion of the lesson is needed at the elementary school level. We feel that the active presentation of information and careful conceptual development are important aspects that are often missing in mathematics lessons. Better ways to describe this portion of the lesson and improved conceptual descriptions of teaching strategies that can be used to enhance development are needed.

In experimental examinations of the development phase of teaching it may be appropriate to give special consideration to multiple learning outcomes. Transfer effects, for example, might be one such variable to measure. In our research, treatment students who were exposed to "more appropriate" development work may have been better able to transfer their knowledge and skills to new situations. This conclusion was suggested by the fact that the treatment group only moderately outperformed the control group on the specially designed content test and *substantially* outperformed the control group on the SRA standardized achievement test. The SRA test clearly required more transfer of ideas than did the content test because the content test was specifically designed to closely match the school's mathematics curriculum. Further and more careful examination of this phenomenon would be useful and interesting. Although the implementation of development was low, and we have noted the need for better measures of developing teaching behavior, we have hypothesized that the development phase was conducted more appropriately in the experimental classrooms than in controls.

Our research on mathematics instruction at the elementary school level has convinced us that teachers do make a difference in student learning and that inservice teachers can be trained in such a way that student performance can be increased. The system of instruction that we see as important can be broadly characterized as *active teaching*. It is important to note that active teaching means different things in different contexts (Good, 1979). For example, older and more mature students can structure more of their learning than can young students.

However, we suspect that the active teaching model we have proposed is also relevant to instruction in secondary schools. More research is needed at the secondary level in order to establish support for this contention and to achieve more understanding of how teachers can structure (without overstructuring) mathematics learning. However, the little research on secondary school mathematics teaching that is available appears to support the advantages of active teaching, at least in terms of short-term student performance. For example, Weber (1978) found that more active teaching in algebra classrooms was associated with more student achievement.

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In a study of seventh- and eighth-grade mathematics classrooms, Evertson, Anderson, and Brophy (1978) found a set of teaching behaviors associated with student learning that are very similar to those included in our training program (Evertson and others, in press; Evertson, Emmer, and Brophy, in press). Among other things, their observational findings indicate that more successful teachers (1) emphasized class discussion, lectures, and drill, and spent less time on individualized techniques; (2) were more task-oriented; and (3) were more active (had more interaction with students; tended to dominate patterns of interaction).

However, as these investigators note, their results are correlations that describe what relatively more effective teachers did during instruction. Whether or not the average teacher would benefit from attempting to become more active is an experimental question. These researchers write, "To run a successful discussion at the junior high school level, the teacher must have good control over the class, enough time to plan the discussion well, and enough energy to do it five times a day. For teachers who lack the classroom control, the time, or the energy, it may well be that assigning seatwork is better than holding disastrous class discussions" (p. 345).

Earlier we noted that elementary school teachers reported positive reactions to our program. We collected this information because we feel that teachers' feelings and beliefs about an instructional program are as important as teacher behaviors. Teacher behavior (whether they used the program in their classroom teaching) is an important comment upon a program, but affective reactions of teachers are equally important. That is, despite the fact that student achievement and attitudes show increases, teachers may choose not to continue an instructional program because it takes more time for preparation, makes the teaching act too demanding, or because it conflicts with teachers' personal definitions of what teaching should be. Thus, we found the anonymous positive feedback of teachers very edifying because it gave a sense of ecological validity to the research.

Clearly, to answer questions about the effects of active mathematics teaching in secondary classes it will be necessary to conduct field experiments and to observe how well an active teaching model can be incorporated into classroom teaching. It will also be necessary to measure the effects of such teaching on student achievement and attitudes. If we are to understand the effects of any instructional program more fully, it may be important to involve teachers directly in the research process. Teachers have demonstrated the capacity to conduct research successfully (see, for example, Behnke and others, 1979), and the need for integrating teacher beliefs into studies of classroom effectiveness has been argued elsewhere (see, for example, Fenstermacher, 1978; and Shulman and Elstein, 1975). Hence, an important next step that we anticipate in our research program is to involve teachers directly in the *design* and modification of the teaching model that we plan to test experimentally in secondary classrooms.

The attainment of a theoretical model that can accurately relate instructional processes to subject matter achievement is a goal that we continue to pursue. Despite the fact that a comprehensive all-inclusive theoretical structure is not yet available, the program has yielded an important set of concepts and interpretable empirical relationships. We feel that the set of concepts that we have characterized collectively as active teaching offers heuristic value to teachers. Hopefully, subsequent research will help to improve the value of our efforts for classroom teachers.

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