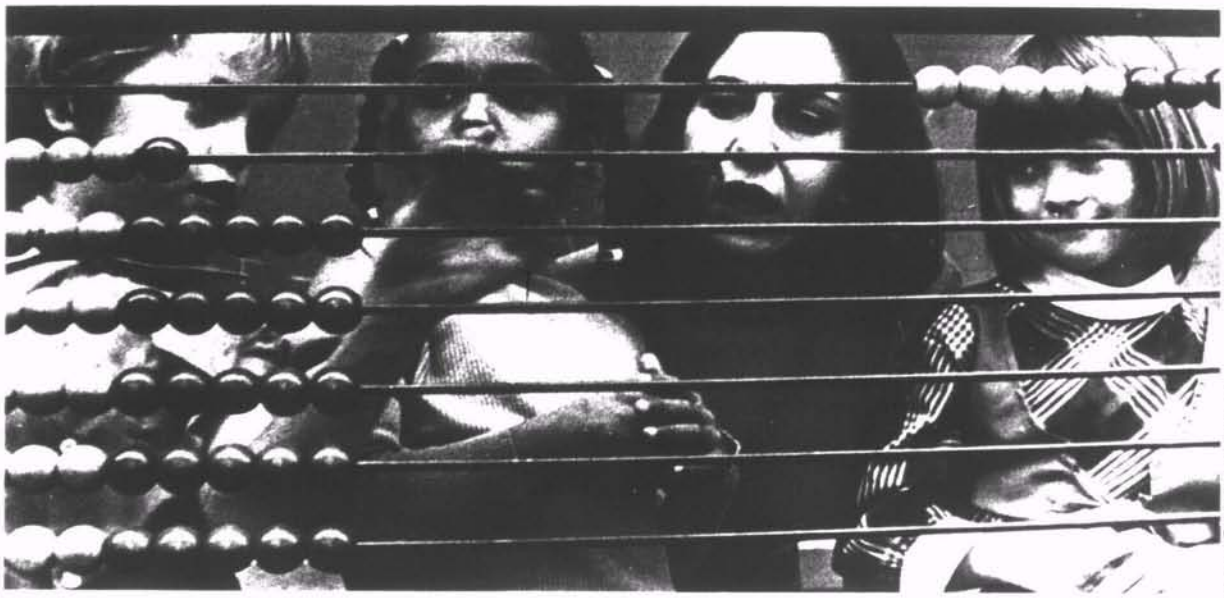

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Students Can Learn to Be Better Problem Solvers

Arthur Whimbey

Students at every level of education from elementary school to medical school are learning problem-solving skills. One of the secrets? "Be careful."

Why are some high school and college students better than others in comprehending and solving problems like this:

> If deleting the letters *r*, *i*, and *e* from the word *surmise* leaves a meaningful three-letter word, circle the first *s* in this word *surmise*. Otherwise circle the *u* in the word *surmise* where it appears for the third time in the exercise.

It is important to know why some students are better than others at solving this type of problem, because it taps the same analytical skills a person needs to master content and solve problems in mathematics, chemistry, or other sciences, and to read tax forms, technical reports, and so much else that is part of a literate and technological culture.

As early as 1950 Benjamin Bloom and Lois Broder (now Greenfield) saw that a major barrier to understanding and teaching problem solving is that thinking is generally done inside a person's head where it is hidden from view. This makes it difficult for a teacher to teach and for a learner to learn. A beginner cannot observe how an expert thinks and solves problems in the way the beginner might observe a golf pro taking a stance, gripping the club, then executing the swing. And the expert cannot observe the beginner practicing, pointing out flaws and showing him/her how to improve.

Thinking Aloud

A way around this barrier is to have people think aloud while they solve problems, so their methods can be observed. When Bloom and Broder had scholastically successful and unsuccessful students think aloud as they solved problems, a striking difference was observed. Unsuccessful students were mentally careless and superficial in solving problems. They often rushed through a problem and selected a wrong answer because they failed to comprehend what was required. When urged to reread the prob-

lem carefully, they often understood it better and proceeded correctly. Even then, however, they were not thorough but settled for a rough interpretation.

Also, they tended to be passive in their thinking. They spent little time considering a question but chose an answer on the basis of a few clues, such as a feeling, an impression, or a guess.

By contrast, successful students made an active attack on problems. When a question was initially unclear, they often employed a lengthy sequential analysis in arriving at an answer. They began with what they understood, drew on other information in their possession, and went through a chain of steps that finally brought them to a solution.

The approach of the unsuccessful student was like that of a novice biology student who hadn't learned to look carefully through a microscope, distinguish the various forms, and put together an accurate picture of the material on the slide.

In order to show students the activities of good problem solvers, Jack Lochhead, director of the Cognitive Development Project at the University of Massachusetts, and I have developed a workbook that includes thinking aloud responses of successful students and professionals. Here is the response of a successful medical student to the problem shown earlier.

Problem Solver's Response

Problem Solver began reading aloud.

If deleting the letters r, i, e, from the word surmise . . .

Problem Solver repeated the letters aloud.

r, i, e.

Problem Solver crossed out the letters *r, i, e* with her pencil. She then read the remaining letters aloud and pronounced the word they formed.

s, u, m, s . . . sums.

Problem Solver resumed reading.

leaves a meaningful three-letter word, circle . . .

Problem Solver stopped reading and thought aloud.

I'm getting to the section where I have to do something if the first part is true. I'm a little confused though, maybe I should start reading the sentence from the beginning. No, I'll read the rest of it.

circle the first s in this word surmise.

I am confused, let me start again.

If deleting the letters r, i, and e from the word surmise leaves a meaningful three-letter word . . .

No, sums is a four-letter word.

Circle the first s in this word surmise.

So I won't do this.

Otherwise circle the u in the word surmise where it appears for the third time in the exercise.

So this is the direction I'm supposed to follow. Let's see, the word surmise appears once . . . twice in the first sentence. Then in the second sentence it appears —three—and that's it. It appears for the third time in the second sentence. So that's the one I should circle.

Now I'll reread the last sentence to make sure I followed the directions correctly.

Otherwise circle the u in the word surmise where it appears for the third time in the exercise.

I'll scan the sentence again to be sure that the word surmise doesn't occur anywhere except the three places I found it. Yes, I circled the right one.[1]

This response illustrates two major characteristics that distinguish successful from unsuccessful students: the step-by-step approach; and carefulness —the concern and quick retracking when ideas become confusing, the rechecking, reviewing, and rereading to be sure that errors haven't crept in, that nothing is overlooked.

Teaching Reasoning Through Discussion

In teaching a skill such as golf, the instructor demonstrates the correct performance and then engages the student in extended, guided practice. To provide a comparable experience in teaching analytical reasoning, educators have shifted away from the frontal, lecture-oriented class to an active, discussion-oriented format. They recognize that while lectures have their place, if teachers lecture for a major portion of the class they are not monitoring and guiding the development of analytical skills.

In the original Bloom and Broder (1950) study, unsuccessful students worked in small groups with a tutor, taking turns solving problems aloud, reading the solutions of successful students, and attempting to correct their weaknesses. They found that by thinking aloud they not only communicated their thoughts to others, but they became more aware of gaps and inadequacies in the way they approached problems. Bloom and Broder observed systematic improvement over the training sessions; students read problems with greater care, and reasoned more actively and accurately. The group also showed a statistically significant gain on the University of Chicago's comprehensive examination, which consists of problems in various academic areas.

The procedure of having students work in pairs or small groups and explain their thinking is now being successfully employed in schools ranging from the elementary level through college. Some classes use the workbook which Lochhead and I developed. In physics and engineering classes at the University of Massachusetts, students spend several hours at the beginning of the semester working in pairs and

[1] Reprinted from A. Whimbey and J. Lochhead, *Problem Solving and Comprehension: A Short Course in Analytical Reasoning* (Philadelphia, Pa.: Franklin Institute Press, 1979).

thinking aloud as they solve problems like the one shown earlier. Then they switch to physics or engineering problems but continue to work in pairs aloud. In contrast, various high schools and colleges will have weaker students spend the entire semester solving such problems. They focus entirely on analytical skills without involving specific content from physics, mathematics, or other subjects.

Project SOAR (Stress On Analytical Reasoning), a five-week pre-first year program at Xavier University in New Orleans, included Piagetian-based science laboratories in the morning, and vocabulary expansion, problem solving, and reading exercises using the thinking aloud format in the afternoon. These gains were reported by SOAR's director:

> The 34 students who initially scored below grade 12 on the comprehension section of the Nelson-Denny Reading Exam showed an average improvement of 1.4 years.
>
> The 43 students who scored below grade 12 on the vocabulary section of the Nelson-Denny gained an average of 1.8 years.
>
> The 21 students who scored less than 70 on the PSAT (equivalent to 700 on the SAT) gained an average of 11.4 points, and the entire group (113 students) gained 7.3 points. These were equivalent to gains of 114 and 73, respectively, on the SAT (Carmichael, 1979, p. 2).

In 1960 a front-page article in *The New York Times* reported an average IQ increase of 10.5 points for the 280 first year students at Whittier College who took a two semester course designed by English professor Albert Upton. The pedagogy of Upton's program was very similar to that of Bloom and Broder. Classes met four times a week; once for lecture and three times in small groups with tutors for problem solving with verbal comprehension and reasoning problems.

Upton's approach has been incorporated into a remedial language arts program called THINK, spanning reading levels one-12, and a parallel math program called Intuitive Mathematics.[2] The problem solving/discussion format used by both programs is proving very effective. In studies by June Gabler (1977), superintendent of the Woodhaven school district in Michigan, THINK produced larger gains on the Nelson-Denny than those produced by stand-

[2] Both programs are available from ISI Think, Inc., 300 Broad St., Stamford, CT 06901.

Problem Solving in Mathematics: National Assessment Results*

An important goal of the mathematics curriculum is to teach students to apply the mathematics they are learning to new or unfamiliar situations. Recently released results from the second mathematics assessment of the National Assessment of Educational Progress (NAEP) indicate that although students are learning many basic algorithmic or computational skills, they have difficulty applying these skills to solve even simple nonroutine problems.

The NAEP mathematics assessment results are based on the performance of a representative sample of over 70,000 9-, 13-, and 17-year olds who took a carefully developed set of about 500 exercises that assessed important objectives of the mathematics curriculum. The results of this assessment represent the best available measure of American students' mathematical achievement.

Students were generally successful in solving simple textbook problems that could be solved by applying a single operation to the numbers given in the problem. However, any exercise that required students to do more than decide whether to add, subtract, multiply, or divide caused considerable difficulty.

At all three age levels, students would frequently attempt to apply a single mathematical operation to whatever numbers were given in a problem rather than analyzing the problem to decide how to solve it. For example, only about 10 percent of the 9-year-olds and 30 percent of the 13-year-olds correctly solved the following problem:

> Mr. Jones put a wire fence all the way around his rectangular garden. The garden is 10 feet long and 6 feet wide. How many feet of fencing did he use?

Almost 60 percent of the 9-year-olds and 40 percent of the 13-year-olds simply added the 6 and the 10.

An important aspect of problem solving is identifying which facts are relevant to a given problem. The following item, which includes extraneous data, illustrates the difficulty students had in analyzing a problem.

> One rabbit eats 2 pounds of food each week. There are 52 weeks in a year. How much food will 5 rabbits eat in a week.

Almost a fourth of the 13-year-olds attempted to incorporate all three numbers given in the problem into their calculation.

When students could identify the appropriate operation, they frequently had difficulty relating the result of their calculation to the given problem. The following exercise involves a simple application of

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ard reading instruction for below average, average, and above average students in both middle and high schools. Also, Intuitive Mathematics resulted in average gains on the Metropolitan Achievement Test which were three times those from standard instruction for high school students with histories of difficulties in mathematics (Gabler, 1977).

Taking a somewhat different tack, Jill Larkin (1977) compared graduate students with outstanding physicists at Berkeley on the way they solved certain physics problems. By having them think aloud she found that the graduate students tended to be "formula bound" while the physicists used mental pictures and other analytical devices to understand situations before starting their computations. Booklets were then developed which successfully taught the analytical strategies to introductory physics classes. In this case the training did not involve small discussion groups; students worked alone, but the booklets did require active responses from the students as they practiced the mental activities reported by the outstanding physicists. Research indicates that small group discussion is useful at all ability levels but is most crucial for the students at the lower levels.

The emphasis on thinking skills is reaching the highest educational settings, as illustrated in these excerpts from a description of a program at McMaster University School of Medicine, written by the director, Howard Barrows (1979).

> The problem solving skills of physicians are central to their ability to apply health care effectively and efficiently. McMaster University School of Medicine has chosen problem based, self directed learning in small groups as the principal format for learning undergraduate medicine. One of the objectives of problem based learning is to help students develop appropriate cognitive skills in medical problem solving. Therefore, a group of us in the School of Medicine completed a five-year study of the problem solving process of the physician.
>
> The next stage of our work was to design appropriate learning units to develop this problem solving process in postgraduate and continuing medical educational settings.
>
> At the present time we are formulating an overall sequence for the development of the problem solving process of the medical student from small group learning around the simulated patient, through individual tasks with simulated patients, ending with structured clinical situations with real patients under pressure to finely tune the problem solving process.

long division. But the answer to the problem is the remainder to the division calculation, not the quotient.

> A man has 1310 baseballs to pack in boxes which hold 24 baseballs each. How many baseballs will be left over after the man has filled as many boxes as he can?

Fewer than 30 percent of the 13-year-olds correctly answered this problem. Over 20 percent gave the quotient as their answer.

Students' mechanical application of computational algorithms often resulted in unreasonable answers that they should have recognized if they had thought about the problem. For example, consider the multiple choice estimation exercise in Figure 1. Many of the students at each age level simply added the numbers in the numerator or denominator. The magnitude of the result is completely unreasonable if one understands what it means to add two fractions less than one. However, rather than estimating the sum, many students appear to have attempted to find some calculation involving the numbers given in the exercise with no concern for the reasonableness of the result. Performance was over 15 percentage points better on a direct computation exercise in which students were asked to calculate the sum of the two fractions.

Figure 1. Sample Estimation Problem

ESTIMATE the answer to $\frac{12}{13} + \frac{7}{8}$.

	Response	Percent Responding Age 13	Age 17
0	1	7	8
0	2	24	37
0	19	28	21
0	21	26	15
0	I don't know	14	16

The results for the exercises described here as well as many other exercises on the assessment strongly suggest that students have become accustomed to mechanically applying mathematical algorithms to problems without analyzing the problems or thinking about the reasonableness of their answers. In recent years, a great deal of attention has focused on the teaching of basic skills, which are frequently equated with routine computational skills. Mathematics skills, however, have little value if they cannot be applied to new or unfamiliar situations.

The ability to analyze a problem situation is as important and as basic a skill as the ability to compute the answer. The assessment results suggest that problem solving is the basic skill most in need of attention in the mathematics curriculum.

Thomas P. Carpenter, Mary Kay Corbitt, Henry Kepner, Mary Montgomery Lindquist, Robert E. Reys.

Problem Solving In Every Class

Many schools are now including a course in problem solving as a part of the curriculum. This is a good start, but one course in problem solving cannot be expected to make students expert thinkers, anymore than a single course in golf or drawing produces experts. Attitudes and activities of analytical reasoning should be reinforced throughout the curriculum.

One example of this reinforcement is the experimental high school program pioneered by Carl Frankenstein of Hebrew University's department of special education (Peleg and Adler, 1977). The Frankenstein program, now incorporated in 42 Israeli schools, instructs teachers in how to engage students in Socratic dialogue. Whether in mathematics, science, or Bible study, whenever students answer questions incorrectly, they are asked to explain their reasoning. This may mean going back to the text to read and interpret pertinent sections or detailing the operations and logic used in reaching an answer. With teachers in all classes focusing on accuracy and social workers giving parents information on providing a supportive home environment, dramatic improvements in cognitive skills have been obtained by Frankenstein and his associates.

Bloomfield College of New Jersey is also stressing mental processes in its first year core program. In describing Bloomfield's program, Hechinger (1979) wrote in *Change* magazine that the "process is the same whether a student is reading a book or doing chemistry—the stress is on systematic analysis and logical examination of problems . . . whatever the subject, there is always a heavy concentration on the analysis of a relatively few pages."

Venezuela has taken such a great interest in the improvement of thinking skills that there is now a Minister of State for the Development of Human Intelligence. More modest efforts can also be effective. A political science class at Bowling Green State University switched from three lectures per week to two lectures and two labs. During the lab hours students read selections from the text and answer questions in the accompanying workbook. They then compare answers with a partner, and where there is disagreement, defend the answer by citing sections from the text and explaining their reasoning. In this way students master the course content while strengthening analytical skills.

Be Careful

Nature enjoys playing an occasional trick on humans, and one of her favorite tricks is to make an important discovery or principle too simple to be recognized as important. However, a growing body of educational research seems to have caught nature at this game. It suggests that one of the most important lessons teachers can impart to students is simply: "Be extremely thorough and careful in your thinking." For years we urged students not to count on their fingers, not to move their lips when they read, and to try to read faster. Research has now shown that there are engineers who count on their fingers, lawyers who move their lips while reading, and literary figures like William Buckley who admit they read "painfully slowly." What we have not done is stressed emphatically to students that the core of academic success is systematic, accurate thought. Newton modestly observed:

> If I have succeeded in my inquiries more than others, I owe it less to any superior strength of mind, than to a habit of patient thinking.

We are now beginning an educational experiment in teaching this "habit of patient thinking." As teachers from the first grade through college begin to echo insistently "Be accurate" and "Explain how you obtained that answer," it will be exciting to see how high we can raise the national levels of reading, mathematics, and problem-solving skills. EL

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