Making Problem Solving Real

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Opening the decade with a call to action, the National Council of Teachers of Mathematics recommended in 1980 that "problem solving should be the focus of school mathematics." The goal of mathematics programs, according to NCTM, is to give students experience in the application of mathematics, which involves selecting and matching strategies to the situation at hand. Problem solving allows students to achieve this end.

Focusing on problem solving requires, as NCTM's *Agenda for Action* suggests, that curricula and textbooks offer a balance of three elements: problem–solving strategies, mathematical concepts, and computational skills. In other words, the emphasis on problem–solving processes should not preclude instruction in mathematical computation and concepts.

Results of the 1980 and 1983 mathematics assessments by the National Assessment of Educational Progress (NAEP) clearly indicated that the focus on basic skills during the 1970s produced students who had mastered computational skills and the routines associated with solving one–step word problems. These students, however, had difficulty with both multi–step problems and problems containing extraneous information. Students often treated nonroutine test problems as if they were one–step problems to which an algorithm could be applied.

Furthermore, Cunningham and Ballew's (1983) study of the strengths and weaknesses displayed by sixth–grade students as they approached solving word

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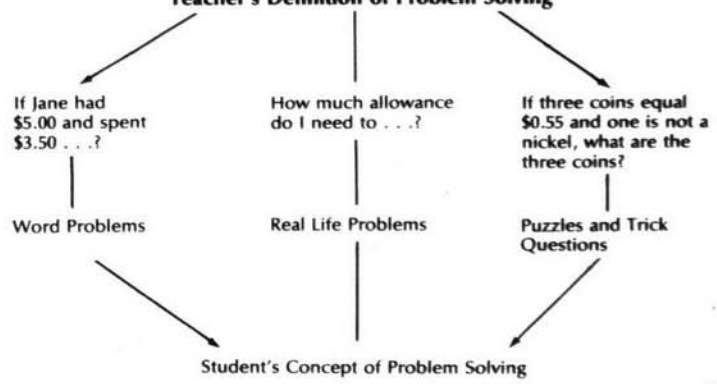
Students will be better problem solvers if the problems are pertinent to their daily lives.

problems revealed that 26 percent of the students lacked the ability to integrate problem interpretation and computation. These investigators posited that the problem–solving process was greater than the sum of its components. Their results implied that mastery of computation and one–step word problems is only productive if students also know how to apply these skills correctly. Such findings focus attention on the need for a broader, more holistic approach to problem–solving instruction.

The *Agenda* also stresses that the use of real–life situations in the teaching process facilitates learning to solve problems independently. This suggestion, however, places a number of responsibilities on the teacher. The teacher must possess the ability to demonstrate and/or guide students toward solutions. This involves helping students see that more than one right answer is plausible, that finding solutions requires time, and that questioning is largely the problem solver's responsibility, not the teacher's. Thus the problems presented must be meaningful to students and worthy of the effort expended in finding a solution.

Figure 1.
Teacher's Definition of Problem Solving



What Do We Want Students to Believe Problem Solving Is?

The ultimate question a teacher needs to consider when planning to teach problem solving is—*what do we want students to believe problem solving is?* This question is crucial in considering what kinds of problems to teach, solution strategies, and methods of presentation. The teacher's response to the question is the basis for the selection of problem–solving activities. Students form a concept or schemata for problem solving depending on the activities to which they are exposed (Figure 1).

For example, if a teacher defines problem solving as the solution of word problems, he or she is likely to teach problem–solving skills this way. Students, in turn, are likely to accept the teacher's definition and associate problem solving with word problems.

Thus, every activity not only teaches students problem–solving skills but also teaches what problem solving is (and is not). Should students believe problem solving is answering word problems? Should they believe it is determining

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answers to puzzles and trick questions? Or should they think of problem solving as making the best choices to obtain a desired goal? We believe the last alternative is the best approach.

If so, instruction must deal with the processes that enable people to obtain something they desire or to rid themselves of something undesirable.

Putting It All Together in the Classroom

The following guidelines may be useful in helping students formulate and solve meaningful problems.

1. *Make sure students understand the definition of and purpose for mastering word problems.* A word problem is composed of brief summary statements describing a problem (the question), data (the numbers), and the context. Problems that refer to students' *physical comfort, security, autonomy,* and *initiative* are the best vehicles to teach the problem–solving process. Problems can relate to:

- Improving school lunch menus
- Regulating classroom temperature in the school building
- Asking for allowance from parents
- Using the telephone at home
- Asking for more time to watch television.

Exploring such problems in depth allows students to make recommendations based on evidence, to influence others to make a decision, to make personal decisions, and to evaluate the decision–making process. Furthermore, a large number of word problems can be generated by students when using real life situations.

One teaching method uses a word problem and then helps students clarify its intent and use. In this procedure, the teacher writes a word problem reflecting a situation that, if improved, would increase student satisfaction at home or school. The teacher's model should include a realistic context, lots of data, and perhaps, but not necessarily, the questions to be answered. For example:

The sixth–grade class at Oak Park Elementary School was unhappy about the food served in the cafeteria. As a means to present their complaints to the principal, an opinion poll was conducted among all students in the school. Students were asked to state their favorite entree. Two hundred thirty–four students selected tacos, 127 hamburgers, 33 spaghetti, 25 pizza, 20 hot dogs, ten chili, and three baked chicken. No one indicated that fish, turkey, soup, or grilled cheese was a favorite entree. Using the teacher's sample summary, students might discuss the following:

- What questions can be answered with this data?
- Where did the data originate?
- How was it collected and organized?
- What other questions are pertinent?
- What other methods could be used to collect data?
- What algorithms are necessary to answer these important questions?
- How can these "answers" assist students in making recommendations?

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2. *Start the questioning; insist on good questions.* In addition to using word problems, the teacher can assist students in formulating appropriate questions upon which to base recommendations with the aid of the reciprocal questioning technique developed by Manzo (1968).

Manzo's procedure, originally used with reading materials, has been modified here to facilitate problem solving in other subject areas. The technique makes the teacher responsible for modeling higher–level questions that require students to collect, organize, and analyze data, and finally, to draw conclusions. The steps of the modified procedure are as follows:

- The teacher provides raw data in the form of word problems, graphs, charts, or newspaper articles featuring statistics and explains the questioning and answering procedure.
- The teacher asks one question and students answer it.
- A student asks a different question and students or the teacher answer the question.
- The process continues as students

and teacher alternately ask and answer questions. (Some questions will require data collection and analysis.)

For example, the teacher's questions could include:

- What conclusions can be drawn from the data describing favorite foods? Why were these conclusions drawn?
- How often should favorite foods be served? How do you know?
- How often can favorite foods be served considering cost and government guidelines?
- Does knowing about students' opinions contribute to a solution? Why or why not?
- Do more students buy lunch when favorite entrees are served? How do you know?
- Which entrees are purchased and eaten and which are thrown away?

3. *Emphasize reasoned responses to problems as opposed to the use of trial and error strategy.* For most students, the processes associated with problem solving are not well-developed. For example, because adults appear to use the trial and error method of problem solving, students may try to do the same. Students, however, often have little idea of how to arrive at a particular solution or even why it was selected from among all the possibilities. The teacher must constantly be aware of this situation and therefore encourage students to support and explain the rationale behind their solutions. This is one way that a teacher can insist that students apply reasoning skills to problem solving.

4. *Emphasize the use of mathematical concepts and computational skills as tools in problem solving.* Our instructional model leads students to answer questions by applying mathematical concepts and skills to data. For example, in devising recommendations for improved cafeteria service, students may find averages, illustrate data with graphs and tables, translate frequencies into percentages, measure portions, determine the amount of wasted food, or determine food costs. In the process, all computational skills from addition to division will be used.

5. *Use the computer where possible.* Students are eager to use an additional tool—the computer. The computer enables the students to analyze data, generate graphs, and print out the final "word problem," complete with recommendations and supportive rationale.

Students must be prepared to use the tools of the information age. Those students who have been *taught* the application of mathematical concepts and are able to make data-based decisions will be better able to benefit from such tools. It is these same students who have developed attitudes that encourage the search for the best possible solution to problems.□

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