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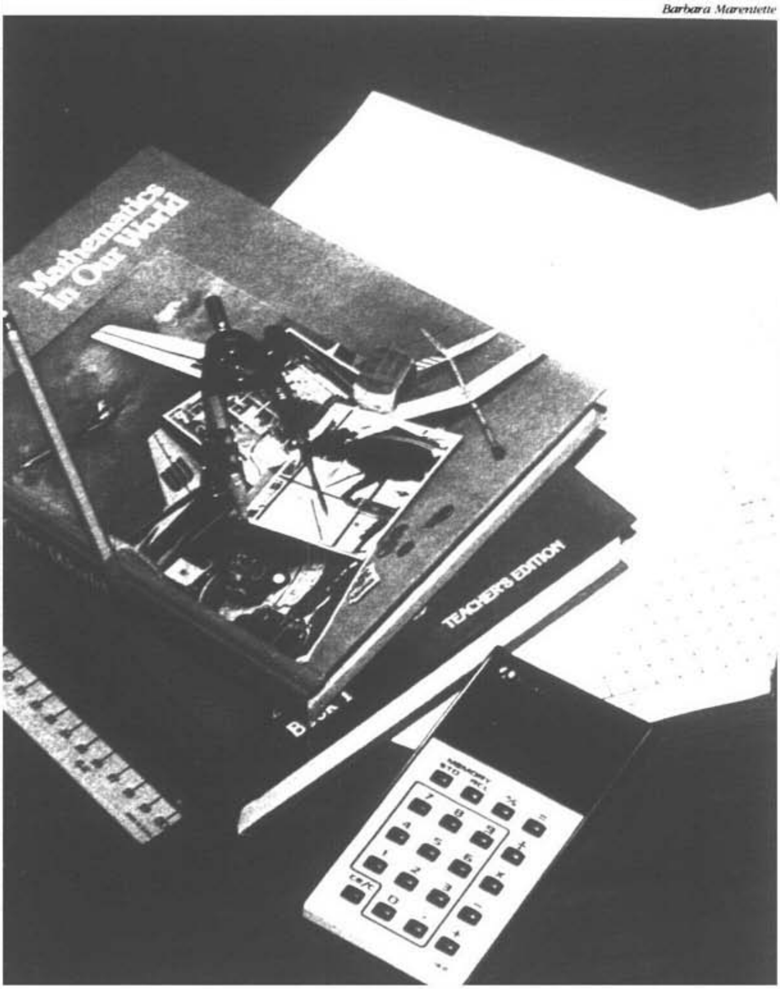
Higher-Order Thinking Skills in Mathematics Textbooks

Despite the lip service that publishers have given to thinking skills, math textbooks have had a poor track record for the last 25 years. Until this situation is rectified, educators are going to have to rely on supplementary methods and materials.

Barbara Marentette



Student acquisition of problem-solving and higher-order thinking skills has long been a goal of schools in general and of mathematics educators specifically. Thus, it is imperative that the tools we use to teach those skills be equal to the challenge of this goal.

In 1984 Steven Willoughby, President of the National Council of Teachers of Mathematics, averred that the "most important factor in determining what mathematics is taught is the textbook used." Unfortunately, mathematics textbooks are not doing enough to actively involve students in the development, practice, and acquisition of higher-order thinking skills.

Analytical Tool and Procedures

Based on a study of earlier schemes for analyzing and evaluating textbooks (Maxwell, 1921; Franzen and Knight, 1922; Fowlkes, 1923; NCTM Committee on Aids for Evaluators of Textbooks, 1965; Eash, 1969; Trueblood and Jansson, 1973), I designed an analysis scheme to overcome some of their weaknesses and to be consistent with the needs of educational evaluators and purchasers. One such need is the availability of descriptive data to use in their decision-making and selection process.

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The analytical tool consists of four lists, which, when accompanied by decision rules, enable analysts to classify printed instructional materials according to type of content, level of cognitive activity, stage of mastery, and mode of response. Because of the interest in higher-order thinking skills, I will highlight only the "cognitive level of student tasks" portion of the analysis system. (See Nicely, 1970, for complete descriptions of the other three components.)

The cognitive level list contains 27 verbs grouped into nine categories and arranged in an ordinal scale (Figure 1). Each of the nine categories (plus an additional category, "No Task; Observe; Read") was assigned a one-digit code, and each verb was specifically defined so that analysts could accurately classify the printed material based on what it actually meant and not necessarily what the textbook author might call it. For instance, an author might use "solve" to describe a variety of behaviors at different levels of complexity.

With assistance from two of my students, I applied the cognitive level list to the "complex numbers" portion of secondary school mathematics textbooks printed between 1961 and 1984, and to the "decimals" portion of elementary (grades 3 through 6) textbooks printed in the mid-1980s. These comprehensive mathematical topics allowed us to examine how the textbook authors would actively involve students at a variety of cognitive levels throughout the stages of readiness, development, practice, demonstration, overlearning, and enrichment, as a mathematical concept was developed in its entirety.

Figure 1. Cognitive Verbs and Codes.

Levels and Codes	Verbs
Level 0	No Task; Observe; Read
Level 1	Recall; Recognize; Repeat; Copy (Imitate, Reproduce)
Level 2	Iterate
Level 3	Compare; Substitute
Level 4	Categorize (Classify, Group); Illustrate (Exemplify)
Level 5	Apply; Relate; Convert (Translate); Symbolize; Summarize (Abstract); Describe
Level 6	Justify (Support); Explain (Interpret); Analyze
Level 7	Hypothesize (Theorize); Synthesize (Organize, Structure); Generalize (Induce); Deduce
Level 8	Prove; Solve; Test (Experiment); Design
Level 9	Evaluate

The textbooks selected were in wide use across the United States and were representative of textbooks that were popular at the time they were printed. Because the National Advisory Committee on Mathematics Education (1975) had reported that students read very little of the textual material in a mathematics textbook, we focused on only those situations in the books where students could be actively involved in their learning and make an overt response. The Committee had found that textbooks were used primarily as a source of problems.

Results of Textbook Analysis

The 1960s. My analysis of the complex numbers portion of the textbooks published in the 1960s revealed that the number of overt responses that required some sort of cognitive process ranged from a low of 210 to a high of 961. The average number of opportunities for students to be actively involved in learning this particular content was 458 instances per book.

In all of these textbooks, 62 percent or more of the problems/tasks were iterative (level 2). The average amount of iterative behaviors per book was slightly more than 75 percent. Relatively few behaviors, other than iterate, appeared more than 10 percent of the time. One book had 18 percent at the application level, while another had 11 percent at that level. All others were significantly lower in terms of application and similarly complex behaviors. Only one book had more than 3 percent hypothesizing and generalization, and several books had no behaviors at that level.

Figure 2. Opportunities for Students to Be Actively Involved in Learning About Complex Numbers in All Textbooks Examined.

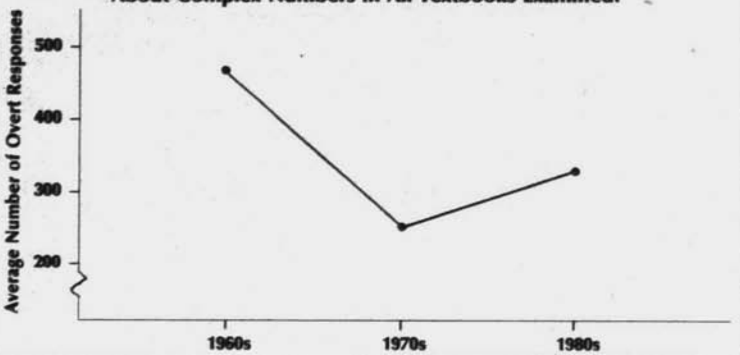


Figure 3. Percent of Lower-Order (1, 2, 3, 4) and Higher-Order (5, 6, 7, 8, 9) Cognitive Behaviors Involved in Learning About Complex Numbers for All Textbooks Examined.

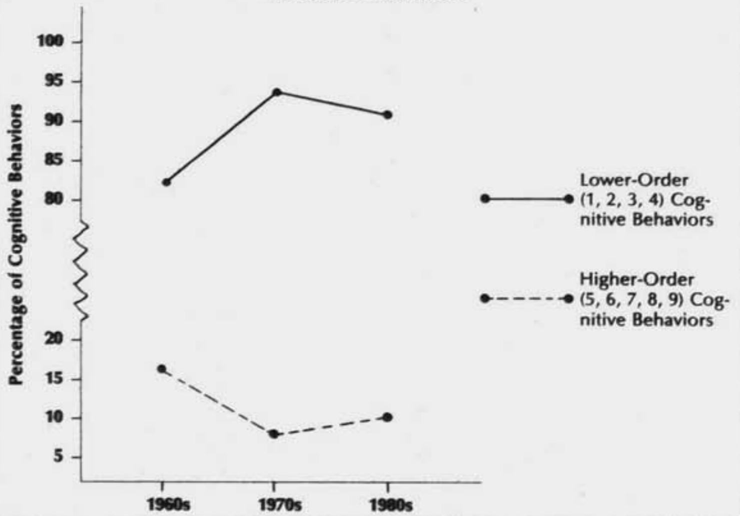
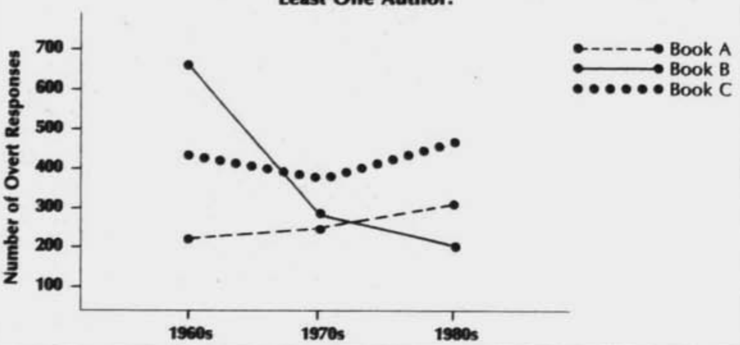


Figure 4. Opportunities for Students to Be Actively Involved in Learning About Complex Numbers in Three Textbooks by the Same Publisher and at Least One Author.



Several textbooks had a variety of (at least seven) cognitive levels, and two had 12 to 15 percent of their total behaviors in the areas of analysis and justification, synthesis and generalization, and proof. In contrast, one book had 98 percent of student behaviors in just one level—iterate. None of the books had student tasks that required evaluative (level 9) behaviors (Nicely, 1970 and 1981).

The 1970s. The analysis of the complex numbers portion of textbooks that were printed in the 1970s revealed that the range of overt responses that required some sort of cognitive process ranged from a low of 112 to a high of 444. The average number of opportunities for students to be actively involved in their learning about complex numbers was 263 instances per book.

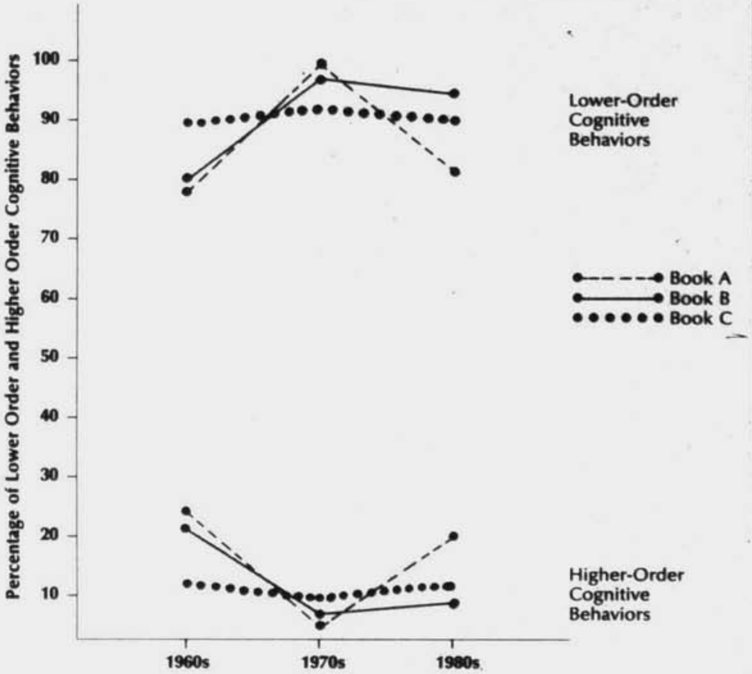
In all of the textbooks, at least 66 percent of the problems/tasks were iterative (level 2). The average amount of iterative behavior per book was 81 percent. In only two books did behaviors other than iterate appear more than 10 percent of the time. These two books devoted 15 to 17 percent of the student tasks to behaviors at the categorizing level. Only two books had many (5 to 8 percent) behaviors at the analysis and justification, synthesis and generalization, and proof levels. One book had 100 percent of student behaviors in the iterate level. Again, no books contained student tasks that required evaluative behaviors (Nicely, 1984).

The 1980s. The analysis of the complex numbers portion of the textbooks published in the mid-1980s revealed that the number of overt responses that required some sort of cognitive process ranged from a low of 216 to a high of 474. The average number of opportunities for students to be actively involved in learning this particular content was 325 instances per book.

In all of the textbooks, at least 77 percent of the problems/tasks were iterative with the average amount of iterative activity more than 86 percent per book. Again, in only two books did behaviors other than iterate appear more than 10 percent of the time. These two books devoted 10 percent and 12 percent of the overt student tasks to behaviors at the prove level. No books contained opportunities for students to operate at the highest level—evaluate (Nicely, Bobango, and Fiber, 1984).

Some trends are apparent across the decades. The *density*—or the number of opportunities for students to be actively involved in learning about complex numbers—decreased substantially from the 1960s to the 1970s and then increased slightly in the 1980s—but not nearly to the extent

Figure 5. Percent of Lower-Order (1, 2, 3, 4) and Higher-Order (5, 6, 7, 8, 9) Cognitive Behaviors Involved in Learning About Complex Numbers in Three Textbooks by the Same Publisher and at Least One Author.



that was evident in the 1960s (Figure 2).

Figure 3 illustrates how the relative emphasis on lower-order (levels 1, 2, 3, and 4) cognitive behaviors increased from about 84 percent in the 1960s to more than 93 percent in the 1970s, and then decreased to 90 percent in the 1980s. There was a concomitant decrease in the emphasis on higher-order (levels 5, 6, 7, and 8) cognitive behaviors—from 16 percent in the 1960s to 7 percent in the 1970s and then to 10 percent in the 1980s. The recent texts show a slight increase, although not a return to the 1960s level.

To determine what happened with a single textbook over time, we examined three textbooks, each of which was published by the same company in all three decades and still had at least one of the original authors. Figures 4 and 5 illustrate the results.

Figure 4 shows that the number of problems in Book A increased in each succeeding time period, while the number of problems requiring student action in Book B decreased substantially from the 1960s to the 1970s and continued to decline slightly into the 1980s. The number of exercises in Book C decreased from the 1960s to 1970s but then increased in the 1980s edition to a level higher than that found in the 1960s. However, it is still lower in the 1980s than either of the other two were in the 1960s.

Figure 5 illustrates that the emphasis on lower-order cognitive behaviors in all three books is similar to the patterns described for all of the books analyzed. In each case, the emphasis on higher-order cognitive behaviors decreased from the 1960s to the 1970s followed by a slight upswing in the 1980s edition. Book A went from 23 percent to 1 percent to 19 percent higher-order behaviors; Book B went from 21 percent to 4 percent to 7 percent; and Book C went from 11 percent to 7 percent to 9 percent. There was a concomitant increase in the number of lower-order behaviors from the 1960s to the 1970s followed by a slight decrease in the 1980s. However, the relative amount of attention paid to higher-order behaviors is still not at the level seen in the 1960s editions.

A new study (Nicely, Fiber, and Bobango, 1984) indicated that recently published elementary school mathematics textbooks also tend to emphasize lower-order cognitive behaviors when teaching decimals. Most of the problems designed for students in the textbook series at the grade levels analyzed were at the iterate level. With one exception—a 3rd grade book in one series—less than 14 percent of the problems were at the application level. All four series offered enrichment activities in grades 4, 5, and 6, but most of these activities would require only iterative behavior on the part of the students.

Comprehensive Mathematics Instruction

The evidence gathered regarding mathematics textbooks supports Bloom's (1984) contention that "teachers in the U.S. typically use textbooks that rarely pose real problems." One corrective measure would be for publishers to employ instructional designers to work with the mathematicians and mathematics educators who write the books. Instructional designers would bring the technical competence needed to develop materials that require problem-solving and other higher-order intellectual skills. The decision to take such steps would be a major financial risk for publishers because they are sensitive to the market and try to provide what the consumers want. The uncertainty lies with what the educational market really wants. Is the attention to problem-solving and higher-order skills just a fad, or is it here to stay?

Are teachers, administrators, and instructional supervisors serious about helping students acquire those important skills? Goodlad asserted that the schools in his study did not appear to be helping students develop those abilities commonly listed as intellectual development and that only "rarely did we find evidence to suggest that instruction was likely to go much beyond mere possession of information" (1983).

If our goal is to help students acquire higher-order thinking skills, then teachers, curriculum committees, and people responsible for staff development will have to supplement the commercially available textbooks as they plan for effective mathematics instruction. Staff development programs (see the September 1984 issue of *Educational Leadership*) can be an asset to teachers and supervisors who value thinking skills, but our curriculum and staff development efforts must be reflected in comprehensive approaches that include an emphasis on both instructional materials and the teaching processes used.☐

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