Trends

Mathematics

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How to Teach Mathematical Problem Solving

"Problem solving" is the "in" phrase for mathematics education this year. And everybody's doing it. Publishers have hired auxiliary teams of authors to write supplementary booklets or special chapters on problem solving. Simple steps for solving simple word problems are now scattered liberally throughout textbooks. Three-week summer institutes to "train teachers to teach problem solving" are springing up like weeds. And they've all missed the point.

Problem solving is not an add-on that can be annexed to an otherwise uninteresting, irrelevant mathematics program (to be deleted when problem solving is no longer in vogue). There is no royal road to problem solving that allows one to become an expert problem solver overnight by memorizing a few rules. And "trained teachers" can no more teach thinking than slaves can teach freedom. (One of the anomalies of American education is that we speak of driver education but teacher training.)

The ability to use mathematical reasoning to solve problems has been crucial throughout the history of mathematics education. It is becoming even more important today because of the increasing availability of electronic devices that can do most of the more pedestrian mathematical tasks more efficiently than human beings could ever hope to. The present commitment to problem solving is simply a reiteration of the reason people have always had for studying mathematics (whether the problems involved commerce, navigation, engineering, astronomy, philosophy, or other matters of interest).

There are many definitions of mathematical problem solving, but most include the following elements:

1. An individual has an unrealized goal that may be indicated only by a vague feeling of uneasiness or by a specific verbalized question.
2. There is some form of blocking that prevents the individual from achieving the goal.
3. The individual uses mathematical reasoning of some sort to bypass the blockage and achieve the goal.

By these criteria, there is no problem without a person. For some people the answer to a question is so obvious that the question is no problem at all. For others, the question may be so far removed from experience or so far beyond their capacity that they have no way to attack it (and perhaps no interest in doing so). For such people the question is not really a problem.

Also by these criteria, most word problems that appear in textbooks are not problems. Usually the student has no inherent interest in the solution and therefore no goal (other than completion of the assignment). Often solutions are either automatic or so far out of the student's reach that they are impossible. Methods children use to solve textbook word problems sometimes show considerable ingenuity and statistical intuition, but are not even remotely associated with the expected method of solution using mathematical reasoning. For example, if the most recent operation studied by the class was multiplication, students often assume (correctly) that every problem on a page of word problems will be solved by multiplication. If the authors have been so unkind as to mix problems requiring several different operations, most students look for a key word that will allow them to choose an operation without the bother of reading and understanding the problem. For example, key words for subtraction include "left," "further," "more," "smaller than," "fewer," and so on. Suppose that immediately after learning subtraction of two-digit numbers in 2nd grade, children are given ten word problems of which nine are solved by subtraction and the tenth is: Mary walked 43 meters north, then 17 meters west. Did she turn right or left? The most common answer will be 26.

How should people learn to solve problems? And how can educators recognize programs that teach people to solve problems? There are no definitive answers to these questions—the subject is too complex—but there is information from research, from experience, and from logical analysis that has been accumulated over the years and is widely accepted by those who have made a serious study of mathematical problem solving.

First, students should abstract mathematics from *their* reality. For very young children reality includes counting fingers, sticks, and other objects; it

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includes simple shapes and relationships. Money, measurements, some pictures, and more complex relationships are real for slightly older children. For progressively older students reality may include still more abstract entities. For all children, a certain amount of fantasy is real and can be used effectively. For generations teachers have known that slower students learn and remember mathematics better when it is related to their reality. But many of these same teachers believed that since quicker students *could* learn to perform calculations without reference to reality, they *should* do so. As a result, many bright students believe there are two distinct kinds of mathematics, the kind that applies to the real world and the kind they are taught in school. Such children tend not to be good mathematical problem solvers.

After abstracting the mathematics from reality and practicing it for awhile, students should apply it to many different situations that seem realistic to them. Properly constructed games can be a good source of problems and practice. Real-world activities such as finding best buys at grocery stores, synchronizing traffic lights to enhance traffic flow, or analyzing news articles provide excellent practice. Well-constructed sets of textbook word problems can also provide good problem-solving experience. A variety of applications is important lest students get the impression that the mathematics they've learned works only with ice cream sticks, or fingers, or money, or some other limited number of situations. One of the great powers of mathematics is that the same mathematical system can be used as a model for a great variety of situations.

Problem solving, like bicycle riding, requires lots of practice. Students should have regular experience, work with others, and formulate and solve their own problems. A small amount of theory about problem solving (such as lists of procedures that sometimes help) can be useful after a lot of practice, but it is of little help before the practice and worse than useless without the practice.

Another characteristic of an effective program for teaching mathematical problem solving is a lot of direct two-way communication between teacher and student and multidirectional discussions with several students and perhaps the teacher. Long lectures are inappropriate even for older students; dull ditto sheets (or a textbook that is essentially hardbound ditto sheets) cannot replace active, interactive teaching using materials, games, and the learners' natural curiosity.

Students should learn to communicate with others about mathematics. They should be able to receive information that is presented orally or in writing. They should be able to present answers orally or in writing so that others can understand them. Textbooks that delete verbs, use strange constructions, or otherwise damage the English language in order to lower the computed "reading level," and teachers who accept numerical answers without any words to provide context, do not help students learn to communicate with others about mathematics.

Mathematical problem solving involves using mathematical reasoning to achieve a desired goal in spite of obstacles. The most effective methods of teaching mathematical problem solving involve always relating the mathematics to the learner's reality and providing plenty of well-motivated practice in solving problems and communicating with others about mathematical problems. Superficial, cosmetic revisions of "back-to-basics" or 1960s "new math" programs will not provide effective means to teach problem solving.

Supervision

ROBERT KRAJEWSKI

Improving School-University Relationships

Seven years ago an ASCD working group on "Roles and Responsibilities of Supervisors" was commissioned to gather current data on the role of the instructional supervisor by reviewing literature and empirical research. Part of the data included five-question interviews with executive directors and other representatives from nine national education associations.[1]

The respondents agreed that interaction among all instructional personnel was central to learning and that clinical supervision is the main process by which instructional supervisors gain access to and understanding of the quality of this interaction. Well over half of the respondents said that instructional supervision should include techniques and practices of clin-

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