Teaching "What to Do" in Arithmetic vs. Teaching "What to Do and Why"

If you want to find out whether students understand the computational procedures they have been taught, ask them. Sandra Nye did.

Photographs by Frank Mantzi

First we X's one times nine. Next we X's the one by the one. Then we go down a line and put a zero under the nine. Next we X's the two by the nine and the two by the one. Now that we have two answers (one on top and one on the bottom). We add them and get an answer of 399.

Once again, the inability of the majority of students to reason accurately has been documented. On the latest mathematical assessment of the National Assessment of Educational Progress (1982), the following question appeared:

Estimate the answer to 3.04 x 5.3.

a. 1.6
b. 16
c. 160
d. 1,600
e. I don't know

The results? Only one-fifth of the 13-year-olds and two-fifths of the 17-year-olds answered the question correctly. These results are disturbing, but they are even more dismal when compared with students' computation abilities. When they were asked to compute the exact answer to a similar decimal multiplication problem, the success rate rose sharply. Almost three-fifths of the 13-year-olds and more than four-fifths of the 17-year-olds were able to compute correctly. This pattern is remarkably consistent throughout the results of the NAEP mathematics assessment—students seem to be able to follow the rules and do the computation, but fail miserably when asked to reason.

Elementary grade children spend an estimated 90 percent of their school mathematics time on paper-and-pencil computation practice, most often learning computation skills by rote. Many students learn the rules and are able to do the computations, but computation success often masks students' lack of understanding and reasoning skill. Stated another way, computation success often masks *our* failure as educators; we have not helped students develop higher-level cognitive skills and understandings that go beyond rote, step-by-step learning. Teaching priorities and teaching pro-

cedures must change. But these changes will occur only if teachers have the support and guidance of administrators who believe that it is not reasonable to make computational proficiency—without equal attention to thinking and reasoning skills—the mainstay of elementary mathematics instruction. What follows are arguments for change put forward by experts with impeccable credentials—elementary grade children.

Teaching Arithmetic for Understanding

Early in December, Sandra Nye, a teacher in Babylon, New York, asked her 4th graders to find the answer to 19 x 21. Sandra had done no instruction in multiplication yet that year, but knew her children had had some experience with multiplication in the 3rd grade. She put the children into groups of four or five students and asked each group to choose a recorder to write down everything they did. As the children worked, Sandra circulated, reading and making notes on their papers to help them go further or to clarify their thoughts. She hoped to gain insight into the children's thinking as they did the computation. What follows are the children's actual reports of the way they solved the problems and their responses to Sandra's continuing questions:

The Red Group

Children: This is how we solved the problem:

$$\begin{array}{r} 19 \\ \times\ 21 \\ \hline 19 \\ 38 \\ \hline 399 \end{array}$$

First we X's one times nine. Next we X's the one by the one. Then we go down a line and put a zero under the nine. Next we X's the two by the nine and the two by the one. Now that we have two answers (one on top and one on bottom) we add them and get an answer of 399.

Nye: Why did you go down a line? What do the two answers mean? Why did you add them?

Children: We changed lines because the rules are to change lines, and the rules are also to put a zero in the beginning of the second line. The rules are that way because there is no way to get 399 with 380 on the same line as 19. We found this out in 3rd grade. We added them because that's the way we were taught, and it gives a sensible answer.

Nye: Now, can you explain each of the numbers in your answer?

Children: The 19 means that one times 19 equals 19. The 38 means that two times 19 equals 38. (Remember it's a rule to put a zero on the second line.) The 399 means 19 plus 380 equals 399. That's how we did this problem.

Nye: *Why* is it a rule to put a zero on the second line? What's the purpose of the rule?

Children: Nobody in our group can remember why we put the zero on the second line. We told you before that it is a rule to put the zero on the second line.

The Blue Group

Children: This is how we solved our problem. First we times 19 x 21 and we got 389, and we found out it was wrong. The teacher put some answers on the board and told us they were wrong. The answers were 57, 3,819, and 389. Then about 5 minutes later we found out the real answer is 399. We added 21 19 times. We proved it by making a 21 x 19 rectangle. The way you set it up is:

$$\begin{array}{r} 19 \\ \times\ 21 \\ \hline 19 \\ 380 \\ \hline 399 \end{array}$$

That is the way you set it up because any time you're timesing two digit numbers, first you put down the first answer and then you put down a 0 on the next line before you multiply.

Nye: How do you get the first number? What does it mean?

Children: You get the first answer by timesing 1 x 19. It means that you have the first part done.

Nye: Why do you "put down a 0" on the next line? What does the number on that line show?

Children: So it makes the number bigger. That line should be the larger number.

Nye: Why should that line be bigger?

Children: So you get a reasonable answer.

Nye: What does 380 show? What numbers do you multiply to get 380?

Nye: Why is it a rule to put a zero in the second line? Children: Nobody in our group can remember why we put the zero on the second line. We told you before that it is a rule to put the zero on the second line.

Students work with base ten materials and gridded paper to figure the answer to 19 × 21.

Children: It shows 38 with a 0 on the end. You multiply 19 x 20 to get 380.

Nye: Where is the 20?

Children: The 20 is the 21 without the one because you times the one already.

The Purple Group

Children: We wrote it this way:

$$\begin{array}{r} 19 \\ \times\ 21 \\ \hline 19 \\ 38 \\ \hline 57 \end{array}$$

Then we found out that we left out the zero in the second number.

Nye: How do you know there should have been a zero in the answer? And how did you realize that your first answer was wrong?

Children: There should be a zero in the second number because when you multiply with tens you put a zero in the ones place because you are not working with the ones. We realized that the answer should be much bigger than 57 because if you put 21 down 19 times it wouldn't add up to 57. It would be much bigger.

Next we wrote down 19 21 times. Then we crossed off two 19's at a time, and put down 38. Then we crossed off two 38's at a time, and put down 76. Next we crossed off two 76's at a time, and we put down 152. It looked like this when we finished.

$$\begin{array}{r} 152 \\ +152 \\ \hline 95 \\ \hline 399 \end{array}$$

Finally we put it like this and it was right.

$$\begin{array}{r} 19 \\ \times\ 21 \\ \hline 19 \\ 380 \\ \hline 399 \end{array}$$

Nye: How did you decide to write it this way?

Children: We figured out the answer with addition and then we figured it out this way.

If they had been asked on a quiz merely to do the computation, the Red Group would most likely have answered it correctly. Yet in both their explanation and in their answers to Nye's questions, they showed that they did not understand the rules they had memorized for the algorithm.

The Blue Group seemed to be furthest along in struggling to make sense out of the partial products. When urged to explain where the 380 came from, they were able to say that it was the result of multiplying 19 x 20. The Purple Group, however, could only justify the final algorithm as being reasonable because it produced the correct answer.

Nye tried a different experiment with the 5th grade class she had the next year. She gave them an individual assignment to complete.

Work this problem, and when you think you have the correct answer, explain why you think the answer is *reasonable*.

Nye had the children attempt this before doing any formal instruction in multiplication. They had, however, been taught the multiplication algorithm the previous year. Nine of the students did the computation accurately, producing one of the two results shown below. Their actual explanations (with misspellings) for why their answers were reasonable are reproduced.

$$\begin{array}{r} 47 \\ \times\ 23 \\ \hline 141 \\ 940 \\ \hline 1{,}081 \end{array} \qquad \begin{array}{r} 47 \\ \times\ 23 \\ \hline 141 \\ 940 \\ \hline 1{,}081 \end{array}$$

My answer was reasonable because I added 141 and 940.

I think my answer is reasonable because I worked it out carefully.

My answer is reasonable because it is right multiplying and adding.

My answer is reasonable because I checked it.

I think it's reasonable because I added the right numbers and timed the right numbers.

I think my answer is reasonable because I took my time and check it.

I think my answer is reasonable because if you estimated 47 it would become 50 and if you estimated 23 it would become

20. 20 x 50 = 1,000 and 1,000 is close to 1,081.

I think my answer is reasonable because 23 is almost 20, and 47 is about 50 and 50 times 20 is 1,000. My answer is about 1,000.

The incorrect responses varied, as did their explanations. Here are some of them:

My answer is reasonable because I know my times tabels, and I was paying attention.

```
   47
 × 23
 ----
  141
 940
 ----
1,161
```

Cause I worked it out four times.

```
   47
 × 23
 ----
  141
 1210
 ----
1,135
```

I think my answer is reasonable because I under stud the problem.

```
   47
 × 23
 ----
  141
   94
 ----
1,135
```

My answer is OK because if you were to take the four and the two and times it you would get eight and the 755 and rounded it you would get 800.

```
   47
 × 23
 ----
  141
  614
 ----
  755
```

I think my answer is reasonable because I did the problem step by step and this is what I came out with 8,281.

```
   47
 × 23
 ----
  141
 8,140
 ----
8,281
```

My answer is reasonable because 141 x 23 is 1,083.

```
   47
 × 23
 ----
  141
 × 00
 ----
1,083
```

Learning rules and being able to apply them are common to many children's experiences in mathematics. They are taught to add the ones first and then the tens in addition, how to borrow in subtraction, to put a zero in the second line in multiplication, to start from the left when doing long division, to multiply across the numerators and denominators to find the product of two fractions, to divide the numerators and denominators of fractions by the same number to simplify them, to line up the decimal points before adding, to count up the decimal places to see where to place it in the answer to a decimal multiplication problem.

But in practicing these procedures, children learn more than steps in a computation process. They learn that if you follow the ruies, it is okay for something not to make sense or, even worse, that it is counterproductive to search for the sense in what you are expected to do.

When children apply the algorithm to an example such as 31 minus 16, it is not uncommon for them to arrive at the incorrect answer of 25. They have "learned" to subtract, but when confronted with a problem that requires regrouping, children will often subtract the smaller number from the larger, something they have been practicing in previous situations where to do so was appropriate. They follow a rule that they have learned, but apply it to a situation it does not fit.

Teaching "What to Do"—A Common Practice

There are many reasons why the practice of teaching mathematics procedures is so detached from what the procedures mean—reasons that must be understood before any widespread changes in mathematics teaching will occur. Here are four of those reasons.

1. *Learning "what to do" is usually easier than learning "what to do and why."* If a goal of mathematics instruction is to get students to produce a page of correct answers, teaching the appropriate procedures will yield this quickly and easily. If no one is pushing for understanding to be taught, why waste time teaching it? There is already more ground than a teacher can cover.

"Elementary grade children spend an estimated 90 percent of their school mathematics time on paper-and-pencil computation practice, most often learning computation skills by rote."

2. *Textbooks emphasize procedures.* The basic goal of textbooks and workbooks is to get students to write correct answers. Teachers' guides urge teachers to teach for the underlying understanding and often provide suggestions for doing so. But the reality is there, implicitly or explicitly: children will show their understanding by being able to complete the work on the page. The writing of correct answers—the primary goal of teaching "what to do"—gets textbook emphasis. Thinking, understanding, and seeing relationships—the primary goal of teaching "what to do and why"—is a low priority.

3. *The pressure of tests looms.* Teachers are accountable for their students' performances and, in some communities, test scores are published in the local newspapers. These tests for the most part test students' proficiency with mathematical procedures. Simply put, there is no payoff for teaching "what to do and why."

4. *Not all teachers understand the difference between teaching procedures and teaching reasoning in arithmetic.* Athough they are proficient with the computational algorithms, many teachers were not taught the reasoning behind them. Teachers cannot teach what they do not truly understand themselves.

Other reasons could be cited as well. Parent pressure often affects what teachers teach. The students make demands, too. When teachers explain the "why" behind procedures they are teaching, they often hear in return, "If you'll just show me how to do it, I'll do it."

The Case for Teaching "What to Do and Why"

In contrast, what are the reasons for teaching arithmetic in the context of meaning and application, of teaching the "why" as well as the "what to do" in mathematics?

1. *When students understand why, their understanding and skills can be more easily applied to new tasks.* Developing understanding requires that students learn skills in the context of

"The writing of correct answers—the primary goal of 'what to do'—gets textbook emphasis. Thinking, understanding, and seeing relationships—the primary goal of teaching 'what to do and why'—is a low priority."

application. Instruction of fractions, for example, can begin by having students investigate how to divide cookies equally. Using paper circles, children "share" six cookies among four people, and then continue with sets of five through one cookie, finding out how much each person gets in each instance.

When children solve this problem, they learn that they can share three cookies among four people in different ways. Some divide each cookie into four parts and give each person one quarter of each cookie. Others share two cookies first, then divide the remaining cookie into four parts. Later, discussing the different ways to solve the problem, children begin to understand why three-fourths is equivalent to one-half plus one-fourth. In this context, the teacher can develop the arithmetic procedure for adding one-half and one-fourth.

2. *Learning what arithmetic procedures mean makes them easier to remember*. When children understand the reasons behind the rules for algorithms, they do not need to store a large number of unrelated rules in their memory. Should they forget a procedure, they have the understanding necessary to reconstruct it.

Using concrete materials for instruction helps students understand and relate the underlying reasons for rules. For example, using models of ones and tens, children learn concretely how to regroup for subtraction as well as when and why regrouping is necessary. To subtract 37 from 60, students construct 60 with six 10s and then regroup one of the 10s as 10 ones to be able to subtract seven ones. After students have practiced this many times, the teacher teaches the procedure for subtraction, relating concrete experience to the standard paper- and-pencil algorithm. In this way students understand the relationship between the algorithm and the structure of our number system.

3. *Learning to reason supports continued learning*. All of us have experienced the joy of accomplishment that comes from figuring out something. So, too, students who are taught to make sense of mathematics begin to understand the relationship among concepts, applying old learning to new learning. As they gradually appreciate the logic of "reasons why," they become increasingly successful in their new learning and increasingly motivated by their own success.

Looking for the Sense in Mathematics

The test of whether mathematics instruction is appropriate is this: can students be successful in the lesson by merely performing mechanically, or must they think and reason? If they need only rote skills, classroom instruction is failing. The alternative is to teach children to apply higher-order thinking skills, to make sense out of what they are being asked to learn, and to understand what they are being asked to do.

The only way to find out if these goals are being met is to ask the ones who really know: the children.□

Marilyn Burns is president, Marilyn Burns Education Associates, 21 Gordon St., Sausalito, CA 94965.