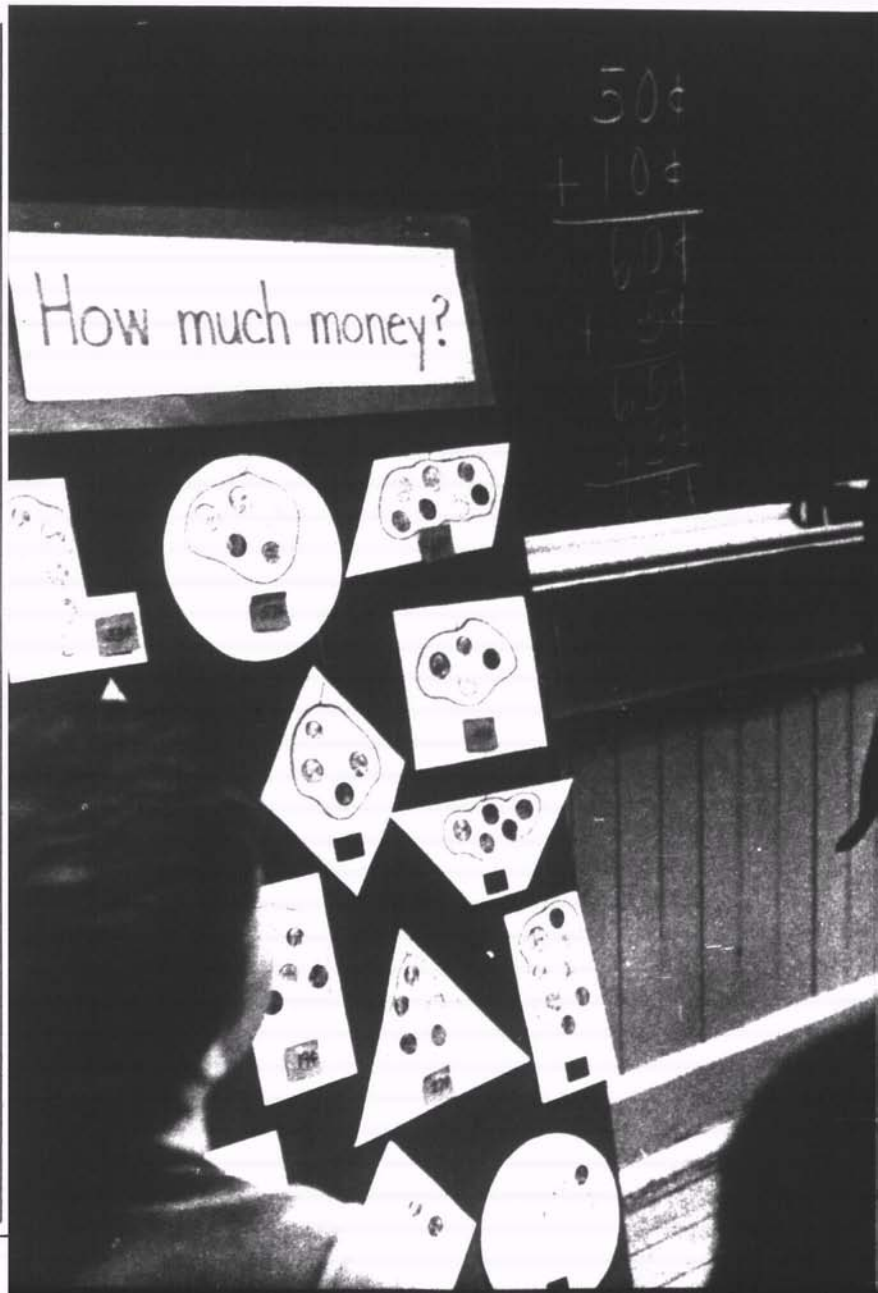
Expertise in Mathematics Teaching

Dorothy Conway is an extraordinary teacher who has taught for more than 20 years in the inner city. The elementary supervisor, the school principal, the parents, and even the students know it. She is something special. Her students really learn math. They love the subject and they are very good at it, not just on tests, but in explaining the intricacies of subtraction, regrouping, or solving word problems—reasoning, in other words.

We have a great teacher. The question is what makes her great or, in the parlance of cognitive psychology, what makes her an expert? At the Learning Research and Development Center of the University of Pittsburgh, we have been working on that question for the last six years. We have learned how she and others *act* as experts. We do not yet know how they got that way.



Defining Expertise; Finding Experts

For the last six years, we have been closely studying the arithmetic teaching of seven expert elementary school teachers, all of whom have taught from 12–25 years. We chose experts because we were convinced that we in the research community had a tremendous amount to learn from how very good teachers approached and carried out their task of teaching. Furthermore, we thought that what we learned might be useful to educational practitioners committed to improving the quality of education and to other researchers involved in studying complex problem solving and other cognitive tasks.

To study experts, we first had to find them. We used the *combined* results of several techniques: (1) we traced students' "growth" scores on standardized tests over a five-year period and picked teachers whose students' growth scores were in the top 15

Research on how expert teachers structure lessons and manage content may suggest how others can communicate mathematics more skillfully and how supervisors can assist them in doing so.

Teacher Dorothy Conway with 2nd grade students at the Brookline Elementary Teacher Center in Pittsburgh

percent for the previous three years; (2) we asked principals and supervisors to review our list and suggest outstanding math teachers; (3) we invited 30 or so of the teachers who had been identified to talk with us about the project and to consider participating; and (4) we visited the classrooms and watched the teachers who had agreed to participate.

The teachers whom we observed taught in schools that were either located in very poor neighborhoods with "difficult" students or in economically depressed areas with high unemployment. We worked with each teacher for a period of one to three years. For purposes of comparison, we have also studied four student teachers in their last year of teacher training using the same techniques.

We conducted our studies by observing, by videotaping between 10 and 25 consecutive topical (subtraction with regrouping or multiplication, etc.) lessons, by having teachers and their students comment on the videotapes, and by extensively interviewing the teachers on their views of pedagogy and math knowledge. Although we have found that not all of the teachers are equally good at all math topics, all are successful in helping their students learn math. Experts are unusually good at constructing series of lessons that successfully transmit the content that needs to be learned. Their lessons are clear, accurate, and rich in example and demonstration of a particular piece of math. The expert teacher presents this new material within a coherent but flexible lesson structure. Both the lesson structure and the content presentation are critical. Further, these lessons take place in an academic environment that focuses on the specifics that students are expected to learn.

Most expert math teachers have goal and action *agendas* that enable them

Photographs by Patricia Crawford

to form lessons that have particular structures with different parts such as homework, drill, review, presentation, guided practice, monitored practice, and tutorials. Experts do not use all structures in each lesson, but they develop specific routines that allow the lesson to progress smoothly and to help set the stage for the particularly important *presentation structure*. Further, experts *explain* new information well.

Novices tend not to have these lesson structures, nor are their routines as effective. For example, novices take up to four minutes to pass out paper; experts take less than 30 seconds. Novices can take as long as 15 minutes (out of 50) to correct the prior day's homework (not discussion, just in-class correction); experts take two or three minutes. Even more important, novices do not know how to convey the important point of a lesson or how to build a rich, comprehensible explanation—experts do.

Embedded Conditions in Expert Teaching

Our research suggests that there are embedded conditions for effective teaching. These conditions have three successively fine-grained levels.

Time and coverage. At the gross level, one set of conditions concerns the time students have available and spend actively engaged in mastering the material, and the scope of the material they cover. Children learn when they spend more time trying to learn material that is directly related to the content to be acquired. Expert elementary math teachers, therefore, arrange conditions so that (1) almost all of the 40 to 55 minutes of a math class are used on math rather than settling down, getting organized, or setting up for a demonstration; (2) they can steal time from blank spaces in the day (for example, number fact drill and stumper problems are good activities for students during lunch, restroom, and bus line-ups); (3) they can borrow extra useful time from out of school by assigning the kind of homework that actually gets done; and (4) they assure students a large number of "painless" practices. Most experts cover at least 40 problems a day orally in games, as jokes, or as written work, and assign another 10 to 20. They do not have students glued to their desks reading illegible dittos of 40 problems. These findings are not new. While it is necessary to use time wisely and to cover material, these conditions are not sufficient to produce knowledgeable students. At a gross level, expert teachers manage their instructional spaces well. If they do not, refining the lesson presentation and choosing great examples rarely make any difference. Novice teachers have so much trouble managing these aspects of teaching that their true skill level at "teaching" is masked. However, managing time and content alone does not make an expert.

Lessons. On a second level, expert teachers also have a good *sense* of a math lesson. Math lessons are qualitatively different from reading, spelling, and social studies lessons. Math lessons have identifiable segments, and while lessons vary, both teachers and students recognize the initiating conditions for the different parts. They are familiar with the *routines* needed to support the lesson segments.

A routine is a cooperative little script of behavior that permits teachers and students to meet shared goals such as passing out papers, doing problems at the board, responding in unison or individually, or asking questions. Routines are the glue of a lesson. Without them, tasks take too long and teachers and students work at cross-purposes. The boundaries between each part of a lesson are clear and brief, and the signals to move from one segment to another are known. Students whose teachers are experts are rarely "lost"; they know what is happening and what they are supposed to be doing. For example, a teacher might move from reviewing a related prior concept such as adding with tens (designed to activate attention on the subarea of math) to introducing the new task goals, such as identifying the kind of subtraction problem that requires regrouping (designed to focus the students' problem solving on a particular task), to teaching the topic by example (demonstrating the concept or procedure), to a public form of practice (preliminary evaluation of lesson success), to a supported practice (a short, independent work session with very rapid feedback), to an extended, less supported practice.

If most of the math lessons have this particular pattern or one similar to it, students learn the pattern and learn their various roles in each segment of it. Each segment can be extended, reduced, or eliminated depending on the content demands of the lesson. For example, introducing fractions in the 3rd grade (where there is little or no procedural requirement) might involve a lesson that is largely manipulative with children working with paper individually or in groups. Introducing the procedure for reducing fractions in the 4th grade might involve a teacher-directed presentation with lots of public student practice. These distinctions between different types of lessons are usually absent from the novice's performance. Thus, experts structure environments to expand opportunities for learning (by increasing time and coverage), and they understand how to conduct a lesson. These structural conditions facilitate learning. The principal, researcher, or supervisor can look for them across grades and math topics.

Content. The next level of embedded conditions for expertise deals with content. Expertise in the teaching of content is *specific*. It deals with how a teacher actually teaches a particular piece of mathematics. The expert teacher introduces subtraction with regrouping, for example, in a way that focuses attention on the problem, its solution, and its applicability. Students come away from the lesson knowing what they can do now that they could not do before and the conditions for using the new material. Recognizing such expertise requires that the observer (or supervisor) know the content very well. To explore this issue in more depth, we will focus on a specific piece of instruction: the teaching of subtraction with regrouping as an example of the content-based specifics of lessons.

To teach a particular content, teachers form content-based *agendas*, sets of goals and supporting actions. Agendas emerge in oral interviews as highly coded note lists. For example, just before teaching subtraction with regrouping, Dorothy Conway said the following:

> First of all I'm going to drill the children on how to add ten to one place numbers. . .then some problems on the board. . .two place subtraction. . .some foolers. . . to warm up. . .then I'm going to

show them using sticks subtraction without regrouping just to get them used to using sticks again. Then I plan to go to the feltboard. . .a little bit more abstract. Then if we have time, I'm going to show them with the real problem. . .straight to the abstract.

Conway's list of actions has implied goals—remembering addition of tens, knowing how to use sticks, following an explanation with felt strips, and finally, working with numbers. There are also several branches with tests for break-offs, such as the "if we have time" statement. For Conway, that means if the students have followed everything and are neither bored nor confused, she will continue. Her agenda is "local" in that it refers only to the specific class period and a specific topic.

Agendas are very topic and time constrained, so phrases like "if I get through the standard form, we may try some word problems; if not, we will go over it tomorrow" often crop up. An agenda almost always contains information on how the topic will be handled: "Well, I've got their tens sticks, and we'll see how to get more ones from them, then the trades on the felt strips, then some written. . ." What the agenda does not contain are the specific examples, page numbers, duration indicators (ten minutes of this, five of that, etc.), nor any activity segment notions (I'll present for ten minutes, then they'll do board work, etc.). Sometimes examples are indicated when large-group movement is planned—games or extensive boardwork. The agenda is a *mental* note pad that guides the teacher from one action to another during the lesson as various goals are met. One label—"some problems at the board"—really stands for an entire sequence of events: selecting children, sending them to the board, selecting problems, monitoring their solutions, selecting correctors, monitoring corrections, and so on. The expert does not need to report that detail because it is all understood as the action gets started.

Novices have agendas too, but theirs are more script-like in that they list all the teacher's moves. Because they are longer, they cannot be recalled as easily during a lesson (novice teachers often lose their place in a lesson). More important, the novice's agenda does not rest on a support system of routines and lesson structures that can be called on during teaching.

When experts introduce a new substantive topic in math, a large portion of their agenda is taken up with the *presentation* of new material. The presentation includes *identifying* the goals (have students recognize, understand, and compute subtraction problems that require regrouping), not just the objective of the lesson but the nature of the inconsistency that requires or motivates the new concept or procedure (using simple column subtraction, you can't take 4 ones from 3 ones); *building representations* of the concept or procedure (counting separate cookies [43 minus 14] does *not* show the problem; tens sticks or felt strips do); *demonstrating* the critical features (it is the *place* value that is problematic, the *unbinding* of sticks, the *trading* of felt strips, and the *rewriting* of tens to ones that matters); *explaining* the how and why of the procedures or concepts (here are the steps); and finally, *proving the legitimacy or consistency of the new idea* with what was known before (after a trade I have the same *quantity* as before).

Conway in Action

We observed Dorothy Conway teach a sequence of eight lessons on subtraction with regrouping. On the first day of the sequence, she quickly reviewed sums with ten (10+4, 10+8, 10+3, etc.) in order to (1) activate students' old knowledge and make it available, and (2) accentuate the base ten aspects of the task or the additive composition of two (or more) digit numbers. Conway then switched to a series of two-digit subtraction problems done at the board with students giving choral support. Finally, she gave them two problems that required regrouping. By rehearsing the simple two-digit column response for subtraction, Conway prevented students from counting up—a clumsy way of getting the answer that does not require regrouping. Specifically, given the problem, 34 minus 17, the students could "count on" from 17, which would focus on the answer, but not on the need for adjusting the algorithm (you cannot take 7 from 4). Conway had students label these last two problems as "foolers." Then she went on to banded sticks, felt strips, and renaming two-digit numbers, each time pausing just before the solution to see if the children could come up with a way of doing it. This clarified and focused on the goal for the series of lessons that would follow.

By the end of the presentation, the children had used sticks to solve regrouping problems and had become skilled at re-expressing two-digit numbers as tens and ones. They learned this through demonstration and rehearsal. Essentially, Conway's first lesson set up the problem and built at least three separate ways for the children mentally to represent it before teaching the solution. Some of the more skilled students whom we interviewed after the first lesson had already seen that "foolers" could be solved by some type of trading or transferring between the two columns. Special skill was involved in using

the banded sticks. With considerable showmanship, Conway had each child select some tens and some ones and then asked a student to give back some (always a number that did not require regrouping—if the child had 4 tens and 6 ones, Patrick would ask for 5 ones, etc.). When she finally sprang a problem, 26 minus 8, on one child, the class was gleeful at the contradiction. Clearly, 26 was more than 8, but the student only had 6 ones. This elaborate focusing would pay off in subsequent lessons when the numerical procedures were explained, rehearsed, and learned. Conway used "concrete manipulative" materials in her example but, more important, she used them for a powerful illustrative purpose, one that would remain accessible to the students for the remaining days of the lesson sequence. This served to construct the first of three representations that Conway used. Expert teachers use good, tight demonstrations that clearly relate to the content being taught.

Still within the first lesson, Conway moved rapidly to a second more abstract example, one that was closer to the eventual algorithm to be taught. The second example involved felt strips. In using felt strips as a demonstration, Conway emphasized two points: (1) trading a ten strip for 10 ones, and (2) retaining the same *value* of the minuend. She *proved* the legitimacy of the trade move to her class of seven-year-olds by showing that the value remained constant. Obviously, not all of the children caught the subtlety of the point; however, many did. The trade operation more closely parallels the numerical regrouping than does the unbinding of sticks, but neither are precisely the same. By the end of the first lesson the students were familiar with three representations (sticks, felt, expanded numbers) but not their connections.

The second and third lessons of the sequence presented the numerical formalisms. By the end of the first day and a half Conway had showed the following: (1) subtraction is both a type of problem and an operation; (2) subtraction problems consist of two types, those that do not require regrouping and those that do; (3) a subtraction operation can be represented as an unbinding of sticks, a trading of felt strips, or a regrouping of numbers; and (4) there are specific procedures for each type of operation. This structure was not suggested by the book or teacher's manual; Conway "invented" it.

"Experts' . . . lessons are clear, accurate, and rich in examples and demonstrations."

Over the next few lessons Conway's students explored subtraction in increasingly complex and ambiguous contexts: word problems, money, with addition and "regular" subtraction, and by correcting other students' work. Note that this practice re-embedded subtraction into the matrix of arithmetic the students already knew, blocking the overuse of the regrouping procedure and thus, recasting the new skill as a now familiar and friendly old one.

What do students get out of such finely tuned teaching? First, the clear presentation of the procedure and the several ways of looking at it provides students with a final skill that is glitch-free, and it provides many of them with a high level of understanding. Second, the rehearsal of both the skill and understanding in many contexts provides the very weakest students with full mastery of the material as well as a list of constraints on when the procedure could not or should not be used. Finally, offering informal proofs repeatedly over the lessons reinforces and develops fairly sophisticated mathematical ideas in the young students.

For the best students, the broad hints in the teaching left them capable of "inventing" all of the next level of work that would not be covered until 3rd grade. Dorothy Conway's lesson sequence is an example of an elegant presentation of new and quite difficult material.

Lessons from Expert Teachers

As researchers, our skill lies in documenting and analyzing the key element in an expert teacher's performance. We can help define expert teaching and perhaps provide new insights into finely tuned teaching that might otherwise be overlooked. Our expertise is not in supervision, but our findings might be of interest to supervisors.

The act of expert teaching is one link in a chain; that is, a good presentation of content will affect students' learning only in the context of a good lesson, which in turn will affect students' performance only if appropriate time is allocated to cover the targeted content in a way that assures the intellectual engagement of students.

Our studies of expert teachers suggest that supervisors might focus on three critical areas of a teacher's repertoire: content knowledge, the patterns that developing lessons take, and academic engagement. Supervisors need to observe both the structural elements of a lesson and the content. To evaluate content, supervisors themselves need considerable subject-matter sophistication to enable them to discuss with teachers detailed ways to introduce a topic, to build better explanations, and to modify sample problems. Supervisors need to be prepared to suggest specific, concrete representations within a lesson to clarify the content. Knowledge of student texts and manuals enables supervisors to support teachers' decisions to depart from a textbook example and explanation in favor of one that the teacher constructs. This knowledge also helps supervisors to construct repairs if such a departure is not successful. In either case, the supervisor's ability to draw on a mental library of specific teaching episodes that work and to become engaged with the content are critical to subsequent dialogue with the teacher.

In examining the teacher's use of effective, clear lesson patterns, the supervisor can be alert to whether the organization of lessons is reasonably transparent, both to the class and to the supervisor. The supervisor can look for a *meaningful pattern* of segments—homework checking and collection, review of terms, presentation of new material, public guided practice, practice, review, and assignment—rather than any *specific* all-purpose sequence. Lessons are necessarily constructed differently; function determines that what would work well in an

exploratory lesson would be unsatisfactory in a review lesson.

While expert teachers tend to be equally expert at managing time and academic engagement, I have not stressed this link in the chain here. This is not because it is not important, but rather because effective management of time and academic engagement is a prior condition for expert teaching: if there is inadequate time to teach or if the students are not paying attention, it doesn't matter how good the lesson is.

Studies of expert teachers do not guide the supervisory process per se, but they do tell us more and more about the content and structure of expert teaching. They suggest that content knowledge is critical, and that supervisors could support content development in mathematics by holding miniseminars on topics such as regrouping, equivalent fractions, multidigit multiplication and division, and word problems. In short, the supervisory process across the discipline and district can inform decisions about inservice education for teachers.

Expert teachers have taught us another lesson. Although expert teachers do many of the same things well, they do not necessarily do them in the same way. In supporting an improving teacher, therefore, the supervisor can attempt to build from the teacher's existing style and suggest ways to modify or tailor these patterns rather than trying to construct an entirely new system. Thereafter, the supervisor's task is to locate where in the teaching repertoire support is most needed—in the structural, management, or content domain—and to see that it is provided.□

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