Complexity in the Classroom

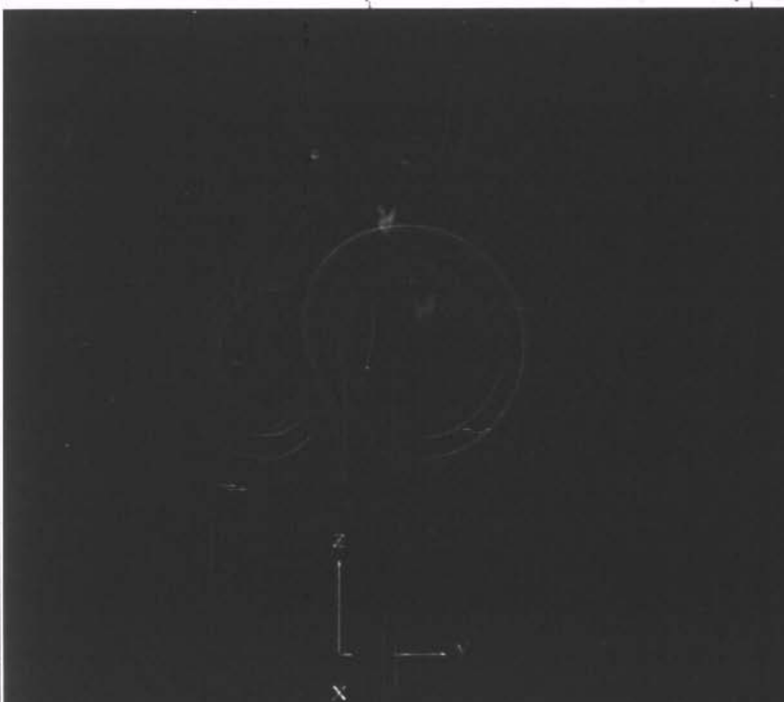
Friday morning math took on a new dimension for 6th graders whose teacher, with a visiting instructor, taught them using insights from *complexity theory*.

To say that tomorrow's world will be complex is to utter the obvious, even the trite. However, I do not use the word *complex* in its usual sense of *more confusion, more decisions* to make, or *more factors* to analyze. Rather, I wish to use it in the sense that it is used in "complexity theory"—a movement in contemporary physics, biology, and mathematics that is encouraging us to look at complexity as a subject in its own right. This movement is changing our views of the origin of the universe, of the way life develops, and of the order we find in both mathematics and nature. Shortly, complexity theory may well have as strong an influence on our views about teaching and learning as it is now having on our understanding of the basic structures of the physical sciences and mathematics.

In Hans Küng's (1988) terms, we are in the midst of an epochal or megaparadigm change, one many label as a move from the abstract formality of modernism to the eclectic creativity of postmodernism (Griffin 1988a, 1988b; Jencks 1987). The change is giving us a new perspective on reality, on how we conceive of order, and on how we solve problems. It is this aspect of postmodernism—the possibility of solving problems aided by the insights of complexity theory—that encouraged me to spend a school year (1987–88) working weekly in a 6th grade mathematics class.

Order and Control

The teacher, Ron Scott, is a former Air Force officer. He is disciplined and runs an ordered classroom. Further, he is committed to helping students acquire their own sense of order and control—over their lives, themselves, their studies. This sense was sorely wanting in most of the students in his class. Ron quickly recognized that the overly simple and linear style of presentation in the mathematics texts the school district used did not help the students develop this order or control. Like most mathematics texts, these encouraged memorization, not understanding; the working of set



Photograph 1: The Lorenz Attractor

Photograph courtesy National Center for Supercomputing Applications, University of Illinois–Champaign algorithms, not the creative utilization of patterns. Ron realized that his own mathematical understanding—stronger than he knew when we began—was based on a knowledge of numerical structure his father had helped him develop when he was a youth. He was also aware that his own way of solving problems and working with numbers was more complex and varied but no less ordered than the methods prescribed in the texts. In his own way, Ron was already moving toward a practical theory of complexity.

We met with students weekly, on Friday mornings for 45 minutes to an hour. We divided them into groups of 2 or 4 and asked them to solve problems that Ron devised—problems that were loosely tied to work they had been doing in science or math class. Our intent was to challenge their own creative and constructive powers; hence the problems were hard, interesting, humorous. They were not the routine drills that filled the textbooks (see "Sample Complex Problems").

Mathematical Chaos Theory

Complexity theory is of recent origin. While no precise date can be set for its inception—indeed to do so would be to oversimplify complexity—a number of occurrences converged in the early- and mid-1970s that gave shape to the theory. One such occurrence was the development of nonlinear mathematics, which, with the aid of powerful computers, allowed mathematicians and other scientists to study directly the complex actions of flags fluttering, smoke curling, water cascading, and cream droplets mixing turbulently in coffee. All of these have high degrees of randomness in them; their patterns are neither fully predictable nor repetitious. The study of these patterns, called *mathematical chaos theory*, is a major force in virtually every science today. It is even emerging, Gleick (1987) argues, as its own interdisciplinary science. This science is, of course, the science of complexity, the science of process.

Nonlinear patterning. What is fascinating about mathematical chaos is not that disorder is present in it but that the disorder present is patterned. Disorder is thus embedded within order—as Paul Davies says, chaos "has some underlying order in its manifestation" (1988, p. 51). To speak, then, of chaotic patterning or ordered chaos is not to speak in contradictions; it is to speak of a newly discovered order, one in which randomness, unpredictability, and indeterminacy are essential features. This type of order is different from that found in teacher's guides or curriculum syllabi.

I will not go into the details of chaotic or nonlinear patterning here. Those interested might begin with Gleick's *Chaos* (1987), Davies' *The Cosmic Blueprint* (1988), or Prigogine and Stengers' *Order Out of Chaos* (1984). I will, though, pick one illustration from chaos theory and show how this guided Ron and me in our work with the students.

Photograph 1 shows what is known as the Lorenz "butterfly." It is a graphic picture of computer data simulating turbulence in long-term weather patterns; it resembles a butterfly's wings or an owl's mask. Although the pattern of the data is random, never following exactly the same line twice, its randomness is patterned; boundaries clearly exist—otherwise the butterfly would have no "wings," the owl no "eyes." The looping back of the data produces the "attractor" area. Ron and I borrowed two features from the chaotic order presented here: (1) boundaries and (2) an "attractor" area.

More dancing, less marching. The concepts of boundaries and attractors within disorder provided a metaphorical basis for us to use as we considered how to interact with the students: we decided to give them flexibility in their intellectual and social organization—they solved the problems in their way and in their time. The patterns that emerged were both disorderly and coherent. Randomness was present in the way they approached their work—they skipped from problem to problem, they left problems unfinished, they interjected social comments into their conversation; they also did all the problems on the page, they went back to the unfinished ones, they put boundaries on their social conversation. Whether an observer saw randomness or progressive order depended on whether that observer was in the class for a few minutes or for the whole class period.

While I was worried about the lack of linear order, about the direct challenge the students were giving to the "time-on-task" maxim, I became aware that over the period of the 45 to 60 minutes a new type of order was emerging—progressive, constructive, personal, interactive. Interest during the class was extraordinarily high, answers emerged from a variety of directions, those who were quiet often raised key questions, and Ron and I virtually never had to admonish students "to finish." While the process seemed disorderly from a segmented view, it had a unity found only by looking at the *whole* class during the *entire* period.

While this insight seems obvious, it runs counter to the reductionist, particularist, and atomistic view that has generally been applied both to science and to teaching. The "new physics" (Davies 1983, Kitchener 1988) is taking a more holistic, time-oriented approach to nature and in doing so is seeing factors in development not seen before. An example from medicine is that the healthy heart beats with a touch of irregularity—chaos. The perfectly balanced heart beat, rhythmical in all respects, occurs a few hours before death and, while not a cause of death, is a sign of its approach. "The

Top-Scoring Class

The 6th grade class described in this article was the top-scoring class in mathematics in the Riverside, California, Unified School District in 1987–88. These results on the California Achievement Profile put the class in the top 25 percent in the state and in the top 3 percent among "schools serving students with similar backgrounds" (that is, same economic and social categories).

healthy heart dances, while the dying organ can merely march" (Browne 1989). I am quite convinced we must re-order our curriculum and instructional methods to promote more dancing and less marching. Both have an order, but one is lively, the other monotonously deadly.

Self-Organization

Another occurrence in the 1970s that has shaped, and is shaping, complexity theory is the concept of self-organization. Self-organization has long been recognized in the biological sciences but not in the physical sciences. As stepchildren, the biological sciences have sublimated their methodologies to those of physics and chemistry. Recently, however, Prigogine (1980, with Stengers 1984) has applied the concept of self-organization to the chemistry of gases, and a number of physicists have applied it to theories about the universe's origin—particularly the "big bang" theory (Davies 1983, 1988). Unfortunately, except for Jean Piaget's work, self-organization has not played a role in curriculum theory or development.

Spontaneous occurrence. In his study of self-organization, Prigogine has shown that it possesses a number of specific characteristics, all of which were useful to Ron and me in teaching the students and in designing a problem-solving environment for them. One of these features is that self-organization occurs suddenly and spontaneously—almost, it seems, out of nowhere. Students "see" at one moment that which they could not see before. As teachers, our task was the tricky one of combining flexible time with directed time in the right proportion to allow and encourage this restructuring to occur. While we cannot specify what this proportion should be, for it varied in our work, we can assert it was essential for us to mix flexible time with directed time.

Occurrence through disequilibrium. A second feature of self-organization is its occurrence only when there is a difficulty to overcome—Prigogine calls these "far-from-equilibrium" situations. It is at this time and under these conditions that internal re-organization takes place. Piaget, himself a biologist, called this *disequilibrium* and made the concept central to his equilibrium-disequilibrium-reequilibration model of development: "However the nonbalance arises, it is the driving force of development .... Without the nonbalance there would not be 'increasing reequilibration'" (1977, p. 13).

It is unfortunate that American educators and curricularists in their early enthusiasm for Piaget paid more attention to his concept of stages and their characteristics than to his concept of how one moves from stage to stage. This latter concept he outlined clearly in such works as *Behavior and Evolution* (1976), *The Development of Thought* (1977), and *Adaptation and Intelligence* (1980). By the time these "biological" works of Piaget appeared, in the late '70s and early '80s, America's love affair with Piaget had passed its peak, and these works were neglected.

For Ron and me, the concept of self-organization through disequilibrium meant we had to organize the Friday curriculum and our presentation of it in such a manner that we had enough of a "burr" to stimulate the

Sample Complex Problems

1. Using only the symbols 2, 3, 12 make at least three mathematical statements. (Equations?)

(Use of exponents and inequalities enlivens this problem.)

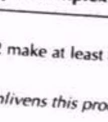
2. Complete the following:

___ ___ ___ 10 20 40 ___ ___ ___
___ ___ ___ 10 20 40 ___ ___ ___

Can you find a way to graph the relationship between these two sets of numerals? What do you see?

(This exercise works well when one set is added-subtracted while the other set is multiplied-divided.)

3. What is the area of the square inside the square?



Now that you know the answer, see if you can find an easier way to solve this problem.

4. A person jogging/running over a mountain pass can average 5 miles per hour. The distance uphill is 15 miles; the distance downhill is 15 miles (how convenient). A person bicycling the same route can move a bicycle up the mountain at 3 miles per hour and down the other side at 30 miles per hour (whee!). If the jogger/runner and the bicycle pusher start at the same time and traverse (?) the same route, which one will arrive at the other side of the mountain first? How about the bottom of the other side? How long will each person take to traverse (that word again) the 30 miles?

(Obviously, the first question cannot be answered without some knowledge of the jogger/runner's speed up the mountain.)

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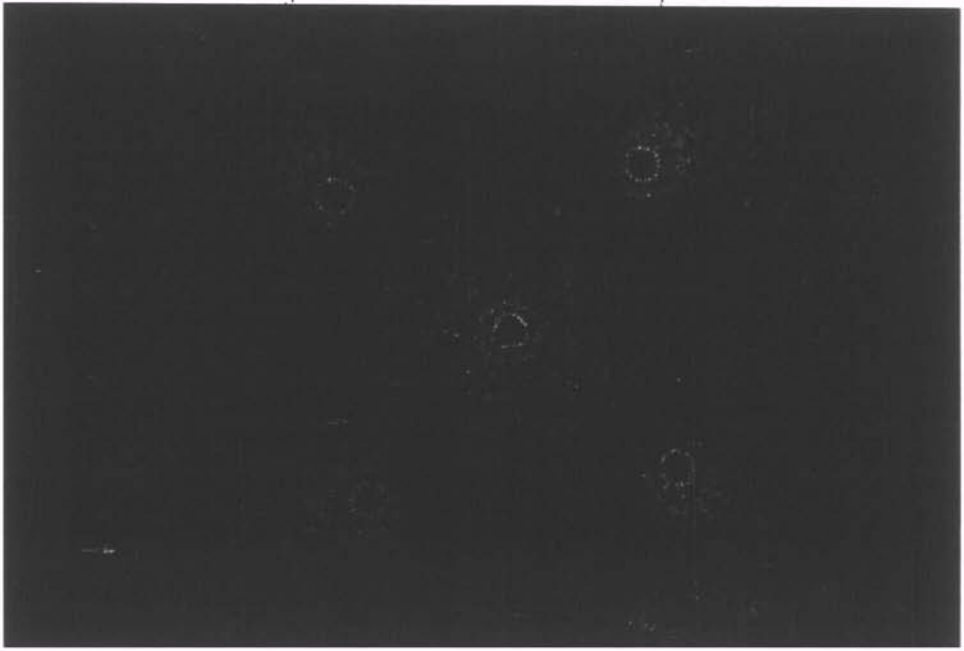
students into rethinking their habitual methods but not so much of a burr that re-organization would fall apart or not be attempted. Maintaining this dynamic tension between challenge and comfort was one of the skills we had to perfect. One device we used was to ask the students to pick out or "unpack" the facts in a problem and then to make up new problems using these facts. A variation on this was to have the students change or rearrange these facts. This device, which proved popular, provided the students with insights into the structure of both the original problem and the ones they constructed.

One conclusion I have developed from this is that problem solving may well hinge on this factor of understanding the structure of the problem; closer reading and strong math skills may be helpful, but they are tangential to the idea of knowing the problem's structure. Too rarely do we teach a problem's structure or even the structure of the operation used to solve the problem. Our methods, if not structureless, at least hide that structure from view.

Occurrence at critical junctures. The third of Prigogine's (1988) characteristics of self-organization revolves around time. He argues that we usually (and incorrectly) look at time in quantitative and cumulative terms, not as we should in qualitative and developmental terms. For him there are critical junctures in periods of development—"bifurcation points"—when re-organization occurs. To find these critical junctures, Ron and I had to analyze time not in terms of amount spent but in terms of the insights developed by the students. This orientation kept the class lively and accounts, I believe, for the quick passage of time in the class. In almost every 45-



Photograph 2: A Mandelbrot Fractal

Photographed by Peter Garrison and reprinted with his permission. This photograph originally appeared on p. 31 in "Glued to the Set," by Peter Garrison in the January-February 1989 issue of Harvard Magazine.

to 60-minute segment we wished for "more time."

In retrospect, one aspect of self-organization we might have utilized better was that of the forced grouping of students. We placed them into groups of 2's and 4's because of my belief in the role cooperation and communication play in re-organization. Now I see we could have had cooperation and communication in a more varied, less forced mode. We should have *allowed* interaction and verbalization to occur, *not forced it*. Learning occurs on a number of levels and in a variety of manners—this is the nature of complexity. Oliver (Oliver and Gershman 1989) makes this point nicely in his distinction between "grounded" and "technical" knowing.

The Application of Fractals

A third occurrence—the last I will deal with in this article—that converged to form complexity theory is of even more recent origin. The discovery and application of fractals (measuring nature's irregularities by using complex numbers) can be tied quite directly to Benoit Mandelbrot's book, *The Fractal Geometry of Nature* (1982). Nature is, of course, highly irregular in many—indeed most—aspects of its structure (coastlines, cloud shapes, star clusters). However, the mathematics we have used to describe and measure nature's irregularity has been highly regular—straight lines, smooth curves, the arithmetic of whole numbers. Our mathematics has thus *approximated* and *abstracted* nature—that is, we use leap years, except for the centennial years not divisible by 400, to compensate for the irregular time of the earth's yearly transit around the sun. The earth does not make this transit in 365 days; not even in 365 days, 5 hours, and 48 minutes. In fact, we cannot measure the transit in such regular terms. Mandelbrot's contribution has been to deal with nature's irregularities in irregular terms.

Photograph 2 shows a graphic of a Mandelbrot fractal—beautiful in its seeming (but random) symmetry. This is actually a computer-generated pattern of a series of numbers—numbers that are squared and added in an a, a^2,

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$a+a^2$, $(a+a^2)^2$ sequence. If one does this with a whole number such as 2—that is, 2, 4, 6, 36, 42, 1764, and so on—the pattern is purely cumulative, growing ever larger. However, if one uses complex numbers—those involving a real number and the root of a negative (i.e., $3 + \sqrt{-2}$)—then strange, unusual, and seemingly symmetrical patterns appear. Some of those patterns, generated by the doubling-adding sequence done thousands of times on a computer, will cluster (like stars) around a certain area or a certain number—an "attractor" area or number. These form the dark spots or blobs in the figure—the "Mandelbrot set," it is called. Other sequences spiral around, gradually or quickly, moving off into higher denominations. The combination of these makes the overall pattern. No quadrant is exactly the same, yet the overall effect of the flames, tendrils, whorls, and filigrees is one of organization in a new sense—complex, chaotic, random organization.

The curricular and instructional task for Ron and me was not to have the students generate chaotic symmetry with computers (although this could be done with advanced classes and advanced equipment) but to help them see that complexity and simplicity are related to each other in such a manner that from simplicity—squaring and adding—complexity can be generated. We believed this would prepare the students not only for the complex world of the future but also for their own work in high school, when the study of complexity might well be part of their science or mathematics curriculum. The device we used was disarmingly simple—generating a set of numbers through doubling (5, 10, 20, 40, 80, etc.) and through halving. Continuing this set was fine practice in mental arithmetic, and it would easily go on to five digits every time we tried it—with, of course, a different first, or seed, number. It is through the halving process that complexity appeared—that is, 5, $2\frac{1}{2}$, $1\frac{1}{4}$, 5/8, 5/16, 5/32, 5/64. This pattern was obviously a challenge for the students. Complexity became even more apparent when we asked the students to put all the foregoing numbers into fourths. The pattern became 20/4, 10/4, 5/4, $2\frac{1}{2}/4$, $1\frac{1}{4}/4$, $\frac{5}{8}/4$, $\frac{5}{16}/4$. Connecting these two together one can see the following:

$$\xleftrightarrow{\text{divided}\quad 2\quad \text{doubled}}$$

$$\begin{matrix} 5/32, & \boxed{\begin{matrix} 5/16 \\ \frac{5/4}{4}, \end{matrix}} & \begin{matrix} 5/8, \\ \frac{5/2}{4}, \end{matrix} & \begin{matrix} 5/4,\ 5/2 \\ 5/4,\ 10/4 \end{matrix} \\ \frac{5/8}{4}, & & & \end{matrix} > 5, 10, 20, 40, 80$$

The students easily saw that 5/4 in both rows was the same, but a new and exciting world opened when they proved that $5/16 = \frac{5}{4}/4 = 1\frac{1}{4}/4$. A deeper understanding of fractions began to emerge. Further, the exercise itself is rich with connections and interconnections. Doing a variety of these exercises, letting the students make up their own patterns, provided excellent drill and substantive insights into the nature of fractions and their relations with whole numbers.

The Complexity of Tomorrow's World

Complexity theory is new. Ron and I used it only on Fridays and only in mathematics problem solving. However, it gave us, along with challenges and complications, insights into teaching and learning we did not have when we began. Some general principles of the theory are universally adaptable and applicable to any subject. The entwining of chaos with order occurs universally and can be seen in all student learning. Self-organization is a feature of all nature, and certainly is evident in students' developing their own creative powers. To see how complexity can emerge from simplicity requires only that we study and teach our subjects—at any level—with depth, not superficially.

As our insights toward complexity deepened and developed, our teaching practices allowed, even encouraged, the students to develop their own appreciation of the irregular but beautiful patterns inherent in both mathematics and nature. The complexity of tomorrow's world is one we hope they will approach not with fear but with awe.□

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