Teaching Mathematics for Tomorrow's World

To prepare students for the future, mathematics teachers must change their curriculums, teaching methods, and assessment techniques.

Today's students will live and work in the 21st century, in an era dominated by computers, by worldwide communication, and by a global economy. Jobs that contribute to this economy will require workers who are prepared to absorb new ideas, to perceive patterns, and to solve unconventional problems. Mathematics is the key to opportunity for these jobs.

Because technology has mathematicized the workplace and statistics have permeated public policy debates, the mathematical sciences are no longer a requirement for future scientists only; they have become an essential ingredient in the education of all Americans. Yet an endless string of reports (e.g., Kirsch and Jungeblut 1986, McKnight et al. 1987, Dossey et al. 1988, Mullis and Jenkins 1988, Paulos 1988, Lapointe et al. 1989) cite serious deficiencies in the mathematical performance of U.S. students. Compared to other nations, we rank very low; compared to our own expectations, we hardly do better. Although basic computational skills are reasonably secure, only 1 in 20 high school graduates can deal competently with problems requiring several successive steps (Dossey et al. 1988).

U.S. students drop out of mathematics at alarming rates, averaging about 50 percent each year after mathematics becomes an elective subject. Blacks, Hispanics, and other minorities drop out at even greater rates (Oaxaca and Reynolds 1988). Because mathematics is a key to leadership in our technological society, uneven preparation in mathematics contributes to unequal opportunity for economic power.

Both economic necessity and concerns of equity demand the revitalization of mathematics education. Job forecasts project a shortfall of well over half a million scientists and engineers by the year 2000; a severe shortage of mathematics and science teachers also seems likely, given increased teacher retirements and rising enrollments in schools. It is, therefore, vitally important both to the nation and to each individual that all students receive a high-quality education in mathematics.

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Goals for Students

For most of the history of education—beginning with Plato's Academy and the Roman quadrivium—students have been required to study mathematics to learn to think clearly. As the embodiment of pure reason, mathematics (especially Euclid) provided an ideal vehicle for teaching rigorous habits of mind.

About 500 years ago, as expanding commerce required widespread use of complex systems of accounting, arithmetic and rudimentary algebra became part of the educational system. Two hundred years ago, "vulgar arithmetic" was a common entrance requirement for the new American universities, while "'rithmetic" became the third "R" in the common expectations for primary education.

Geometry and arithmetic—thinking and calculating—are not only paradigms of school mathematics but also caricatures of mathematics in the minds of parents. Today, for quite different reasons, neither goal is especially relevant. Although most children learn to calculate well enough, calculators have made this hard-to-learn skill virtually obsolete. And although high school students still study proofs in geometry, little learned there—and little is all it is—transfers to clarity of thought in other areas of life.

To help today's students prepare for tomorrow's world, the goals of school mathematics must be appropriate for the demands of a global economy in an age of information. The National Council of Teachers of Mathematics' new *Standards for School Mathematics* (NCTM 1989) identifies five broad goals required to meet students' mathematical needs for the 21st century:

- *To value mathematics.* Students must recognize the varied roles played by mathematics in society, from accounting and finance to scientific research, from public policy debates to market research and political polls. Students' experiences in school must bring them to believe that mathematics has value for them, so they will have the incentive to continue studying mathematics as long as they are in school.

- *To reason mathematically.* Mathematics is, above all else, a habit of mind that helps clarify complex situations. Students must learn to gather evidence, to make conjectures, to formulate models, to invent counterexamples, and to build sound arguments. In so doing, they will develop the informed skepticism and sharp insight for which the mathematical perspective is valued by society.

- *To communicate mathematics.* Learning to read, to write, and to speak about mathematical topics is essential not only as an objective in itself—in order that knowledge learned can be effectively used—but also as a strategy for understanding. There is no better way to learn mathematics than by working in groups, by teaching mathematics to one another, by arguing about strategies, and by expressing arguments carefully in written form.

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- *To solve problems.* Industry expects school graduates to be able to use a wide variety of mathematical methods to solve problems. Students must, therefore, experience a variety of problems—variety in context, in length, in difficulty, and in methods. They must learn to recast vague problems in a form amenable to analysis; to select appropriate strategies for solving problems; to recognize and formulate several solutions when that is appropriate; and to work with others in reaching consensus on solutions that are effective as well as logical.

- *To develop confidence.* The ability of individuals to cope with the mathematical demands of everyday life—as employees, as parents, and as citizens—depends on the attitudes toward mathematics conveyed by school experiences. One of the paradoxes of our age is the spectacle of parents who recognize the importance of mathematics yet boast of their own mathematical incompetence. Mathematics can neither be learned nor used unless it is supported by self-confidence built on success.

Curricular Change

Even when measured against older standards, the prevailing mathematics pattern in U.S. schools is an "underachieving" curriculum (McKnight et al. 1987). "We have inherited a mathematics curriculum conforming to the past, blind to the future, and bound by a tradition of minimum expectations" (NRC 1989).

When compared to the NCTM's five goals, today's curriculum is totally inadequate. The new curriculum standards (NCTM 1989) make clear that the whole environment of learning must change: not only what is taught but also how it is taught and how it is assessed.

Recent studies of mathematics education (e.g., McKnight et al. 1987, NRC 1989, AAAS 1989, NCTM 1989) reveal principles required for effective learning. While different reports stress certain aspects (such as international comparisons or the impact of computers) more than others, there is widespread—and perhaps surprising—agreement on certain necessary actions:

- *Raise expectations.* Evidence from other countries as well as from some U.S. districts shows that if more is expected in mathematics education, more will be achieved. Despite the common belief of U.S. parents that special talent is required to succeed in mathematics, in reality all that is required is hard work and self-confidence. Children can succeed in mathematics, and they will succeed if we expect them to.

- *Increase breadth.* The traditional mathematics curriculum focuses too narrowly on a few topics of limited appeal and utility—on arithmetic, which leads to algebra, which in turn leads to calculus. Most students would benefit from a curriculum with a broader vision, one that reflects the expanding power and richness of the mathematical sciences. Estimation, chance, measurement, symmetry, data, algorithms, and visual representation are as much part of mathematics as computation and manipulation—and for many students, they are more interesting.

• *Use calculators*. Nothing better symbolizes the backward nature of our mathematics curriculum than the reluctance of teachers and test makers to make full and appropriate use of calculators. Research shows that appropriate use of calculators enhances both children's understanding of arithmetic and their mastery of basic skills. It is more important for children to develop good number sense than merely to memorize methods of calculation. Besides offering an important method of calculation (others being mental arithmetic, estimation, paper-and-pencil, and computers), calculators provide a powerful tool for developing number sense.

• *Engage students*. Research in learning has demonstrated repeatedly, in a variety of ways, that students do not simply learn what is taught. Rather, their experiences modify prior beliefs, yielding a mathematical knowledge that is uniquely personal. Clear presentations alone are insufficient to modify students' misconceptions. To ensure effective learning, mathematics teachers must employ classroom strategies that make students active participants in their own learning rather than passive receivers of knowledge.

• *Encourage teamwork*. Employers repeatedly stress the importance of being able to work with a team on common objectives. Most complex problems demand the talents of many different people. Students of mathematics must learn how to work with others to achieve a common goal: to plan, to discuss, to compromise, to question, and to organize. Teamwork in the classroom not only teaches these skills, it is a very effective way to learn mathematics—by communication with peers.

• *Assess objectives*. Assessment is an issue of increasing importance in American education. To be effective, assessment must be aligned with the objectives of learning. When assessment is dominated by standardized multiple-choice tests, as it is now, teachers teach skills required for those tests regardless of what the official objectives of instruction may be. Assessment must become an integral part of learning, not just an infrequent objective exam; it must be designed to reflect what students know and how they think. Above all else, assessment must align with curricular goals: to value mathematics, to reason mathematically, to communicate about mathematics, to solve problems, and to develop self-confidence.

• *Require mathematics*. Students should study mathematics every year they are in school. Projections of future jobs as well as patterns of college course prerequisites show a consistent increase in the mathematical demands of employment and careers. There is no point at which a high school student can correctly conclude that he or she needs no more mathematics. All students who are college-bound need four years of mathematics to meet college course prerequisites. Students who are not preparing for college must keep up their mathematics skills in anticipation of vocational or on-the-job training. Regardless of their career goals, all students should study the same core of broadly useful mathematics while in high school.

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• *Demonstrate connections*. The power of mathematics derives both from its internal unity and its external applicability. Everything is connected. Results in number theory provide clues to problems in geometry and are applied in computer science and satellite engineering. Students must see these connections at every opportunity in their school experience. Connections motivate learning and reinforce ideas arising in different contexts. We can no longer afford to let mathematics remain an isolated discipline nor to permit continued fragmentation within the mathematics curriculum itself into isolated courses, separate topics, and disconnected bits of knowledge.

• *Stimulate creativity*. Too often mathematics is judged "dull" by students, even by very good students, because teachers, textbooks, and tests insist that each problem must be solved by one proper method yielding a single correct answer. Nothing could be further from the reality of mathematics in practice. Multiple approaches, invention of new methods, and varieties of solutions are far more typical than are automated answers. Computers and calculators now perform most of the routine tasks of mathematics. In a computer age, one needs to use one's imagination as much as one's intellect, one's judgment as much as one's memory.

• *Reduce fragmentation*. Curriculum planning based on specific learning objectives has produced an atomized curriculum of particular techniques practiced on problems specially selected to illustrate textbook methods. Real problems don't come in compartmentalized form. In school, the best clue concerning the approach to a problem—and the approach is in many cases the most important decision—is which section of the book it appears in. Fragmenting the mathematics curriculum destroys the logical unity of mathematics that is the primary source of its unique power to model the world.

• *Require writing*. Nothing helps a student learn a subject better than the discipline of writing about it. Writing in a mathematics class serves several purposes. It advances the goal of learning to communicate about mathematics; it helps students clarify their own understanding as they try to put ideas into coherent written form; and it provides an opportunity for students who like writing better than mathematical abstraction to grow in the discipline with a vehicle more suited to their abilities. Many teachers have reported positive results from journal writing and other "meta-assignments" in which students reflect on their experiences in learning mathematics (Connolly and Vilardi 1989). In contrast to the typical school ritual of mindless mimicry calculations, writing enhances learning by involving the student in expression of meaning.

• *Encourage discussion*. Most talk in a mathematics class comes from the teacher, not the students. In a typical class, students take notes, practice what the teacher has demonstrated, and then work in isolation to perfect the technique. None of this engages the student's mind as effectively as does vigorous argument and discussion. Argument in search of convincing proof is the essence of the mathematical method. It can be learned only by doing, not by listening.

For Information: *Everybody Counts*

The National Research Council has published *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (1989) as a "public preface" to the work of three NRC units (the Mathematical Sciences Education Board, the Board on Mathematical Sciences, and the Committee on the Mathematical Sciences in the Year 2000) in revitalizing mathematics education. *Everybody Counts* describes forces that affect mathematics education—computers, research, demography, competitiveness—and explains how each is a major force for change, while interactions among these forces produce a system that is peculiarly resistant to change. The book describes specific changes that our nation's schools and colleges must make in their mathematics programs if they are to meet the needs of students and the country. Available from the National Academy Press, 2101 Constitution Ave., N.W., Washington, DC 20418. The price of one copy is $7.95; 2–9 copies can be purchased for $6.50 each; 10 or more copies, for $4.95 each. Orders must be prepaid.

Changing Emphases

Curriculum. Any change in curriculum will require significant, specific change in content. Both the role of computers and the increased applications of mathematics have made certain parts of mathematics more important and others less so. Many areas of mathematics that are commonly used in both civic and practical contexts are rarely taught in school, while other topics that have long since outlived their usefulness remain in the curriculum simply because they are still on tests or in texts.

In a revitalized school program, many widely used areas of mathematics must receive increased emphasis:

- geometry and measurement
- probability and statistics
- patterns and relationships
- spatial reasoning
- collecting data
- observation and conjectures
- estimation and mental arithmetic
- genuine problems
- three-dimensional geometry
- graphical reasoning
- discrete mathematics

Several other topics, which now consume a major part of the school mathematics experience, should be reduced dramatically:

- fractions
- long division
- graphing by hand
- paper-and-pencil algorithms
- two-column proofs

The first four of these items have become less important because calculators and computers are both more accurate and reliable than manual calculations. The last item, two-column proofs in geometry, has never been part of real mathematics: it exists only in school geometry as an exercise, totally isolated from the rich reasoning so appropriate to geometrical intuition. Geometry can be taught more effectively without this stereotyped form of proof, and proofs can be taught more effectively in contexts besides geometry.

These changed emphases must be implemented in a way that builds more integrated mathematical experiences from primary school through high school. Major themes of mathematics such as chance and change, shape and dimension, quantity and variable should run through the entire curriculum, woven into a single fabric of mathematical method.

Teaching. Just as content changes, so too must teaching methods. What is taught matters little if students are not provided with suitable opportunities to learn. Effective classroom practice will emphasize:

- active learning
- problem solving
- concrete materials
- instructional variety
- oral communication
- written exercises
- paragraph answers
- continual assessment

At the same time, many common practices must be minimized, as the evidence shows conclusively that they are not particularly effective (Cooney 1988):

- teaching by telling
- rote memorization
- one method, one answer
- memorizing rules
- template exercises
- routine worksheets

Testing. Finally, testing must change. No effort to change content or teaching practices will succeed unless the instruments of assessment match curricular objectives. Effective assessment will:

• be open-ended, not just multiple choice;

• allow calculators in virtually every context;

Curriculum, teaching, and testing must change together to improve mathematics education. Unless all improve in concert, nothing will change.

- provide opportunities for students to show what they know and how they think, not just seek to determine what they do not know;
- emphasize integration of knowledge and holistic strategies for approaching problems (for example, estimation, graphing, models, computers, calculation);
- be integrated with teaching, not separate from it;
- employ a variety of methods, including observation, oral protocols, student notebooks, written tests, and group projects.

Commitment to Change

Curriculum, teaching, and testing must change together to improve mathematics education. Unless all improve in concert, nothing will change. The NCTM *Standards* provides a clear blueprint for reconstructing U.S. mathematics education. We know what needs to be done, and we know how to do it. What's required now is a commitment to action.□

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