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Transforming Mathematics Education

NCTM's *Standards*, which provides a framework for strengthening both the teaching of mathematics and the content of the curriculum, promises to fulfill a new vision of mathematics education.

Mathematics education in American schools is entering a vitally important era of transformation in both teaching and learning, which has been called for by myriad reports (National Commission on Excellence in Education 1983, McKnight et al. 1987, Dossey et al. 1988, and Steen 1989). Documented in these studies and analyses are the shortcomings of the curriculum, the lack of parental support, the national distaste for mathematics, and other factors impeding progress in mathematics education.

The National Council of Teachers of Mathematics (1989) has responded to these challenges with its *Curriculum and Evaluation Standards for School Mathematics*. This document outlines the broad curricular goals that our nation's schools should be striving to reach and suggests methods by which progress toward those goals should be assessed at the student and school levels. This vision for school mathematics is predicated on several fundamental changes in how mathematics is viewed by our society and how it is taught in our schools.

Pursuing this vision will require a major effort to change the public's conception of mathematics and the ways in which this perception is passed to the youth of the nation. Mathematics must come to be seen as a *helping discipline*, not as a subject area that sorts and rejects students on their inabilities to perform. The *Standards* also calls for a change in the teaching of mathematics from students' passively receiving instruction to their active involvement, supported by technology, in investigating numerical, spatial, and data-related situations. If the total effort is to result in lasting positive change, these transformations of perceptions and teaching methods must be accompanied by concomitant changes in curriculums and assessment.

Changes in Teaching

Historically, the teaching of mathematics has taken place in an atmosphere of rigidity and student fear, as the accumulated knowledge of past generations has been transmitted to anxious students in classrooms void of active, engaged investigation. The *Standards* calls instead for a curriculum that provides for students' participation, across the grades, in *constructing* their conception of mathematics. This construction must involve students in "doing mathematics" with manipulatives, discussing the results of their investigations, and writing the results of their experiences. Such experiences, at each grade level, allow students to build on their extant knowledge by inventing new methods of assaulting problems (Resnick 1987). While providing students ample opportunities to grow in their ability to apply mathematics, such approaches will also develop confidence in their ability to succeed in mathematics.

Some of the changes recommended by NCTM will require additional teacher preparation. The new image of mathematics teaching envisioned in the *Standards* will ask teachers to become more proficient in guiding small-group work in the classroom and in managing student discussion and record keeping in math settings (Davidson 1989). Teachers will need to become co-investigators in the process of learning and structuring mathematics and in creating rich environments that allow for investigation and growth, while providing students ample room to solidify their previous learnings in making connections to new content. Time for these investigations can be found by reducing the time previously spent on over-practicing computational procedures; for example, long division and fraction operations in K-8, factoring in Algebra I, excessive congruence proofs in Geometry, and computation with logarithm and trigonometry tables in Algebra II and Trigonometry.

The development of such questions and settings will require continuing professional development tied to specific grade/age ranges in the curriculum. In addition, it will require that class loads for mathematics, as for language arts in the upper grades, be kept at levels that allow for lively student-teacher interaction. Teachers, as well as students, need to learn to function in an atmosphere that considers meaningful "What if . . . " questions (see "Examples of 'What if . . .' Mathematics Questions," p. 24).

"What if . . . " questions ask students to apply their mathematical knowledge, while giving them ample opportunities to select and construct their own methods for reaching solutions. These methods can be discussed and refined, as needed, to develop the more conventional methods. In the past, the rush to move students to the "one way" to approach these types of problems has shortchanged those who were not developmentally ready for more formal, symbolic mathematics. This strategy has also deprived students of the opportunity to construct their own approaches, which can later be modified to more efficient algorithms.

Approaches to the teaching of mathematics that rely on student construction of knowledge and procedures call for new methods of assessment as well. The *Standards* outlines several important guidelines for measuring students' growth, particularly their ability to reason, solve problems, and communicate their mathematical analysis of situations. Observations, interviews, student journals, and other avenues of assessment provide exciting possibilities for collecting data that provide different views of the nature of student understanding of mathematics. These more "authentic" methods often disclose student misconceptions missed by traditional assessments. Success in changing evaluation methods leads to further opportunities to strengthen teaching methods.

Changing the Nature of Content

The changes brought to mathematics by the advent of computer technology have sharpened the need for students to develop strong conceptual knowledge. The need for extended skill in computation with numbers having many digits or extraordinary fractions has diminished due to the availability of technology that does these computations quickly and quite accurately. The important question has become, "What computations, if any, need to be made in a problem situation?"

The overemphasis on procedural skills in mathematics has often acted like a filter, removing capable thinkers from our mathematics classrooms. The emerging curriculum is much stronger in conceptual knowledge, while still stressing the importance of basic facts and operations. For example, in Algebra I, over nine weeks is usually spent on the factoring to solve equations such as $6x^2 + 15x + 6 = 0$. Students who are unable to do this rarely ever see the use of algebra in application settings. The graphing

Fig. 1. A Graph of the Equation $6x^2 + 15x + 6 = 0$ Performed on a Computer

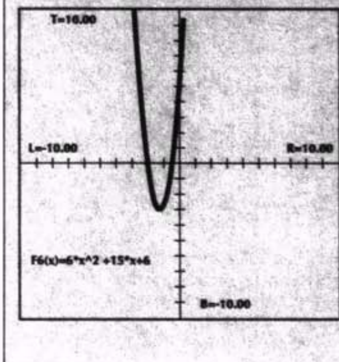
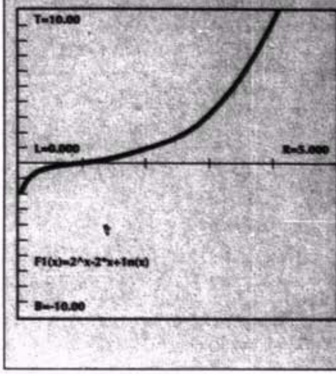


Fig. 2. A Graph of the Equation $f(x) = 2^x - 2x + 1n(x)$ Performed on a Computer



hand calculator and readily available software, however, quickly provide views of the equation that allow for graphical analyses (see fig. 1).

Figure 1 shows the results of using a software package (Waits and Demana 1988) to graph the related function. The solutions to the equation are the points where the graph crosses the *x*-axis. In this case, it is easy to determine and check that these values are -0.5 and -2. If the values had been harder to determine, the student who performed the operation could have used the *zoom* feature to enlarge the region around the point -0.5 on the graph for a closer look. The values *T* and *B* on the screen give the *y*-values for the top and bottom of the screen, the *R* and *L* values give the corresponding information about the *x*-values for the right and left of the screen. The student can carry out this operation without any special factoring skills. Should she have needed to find the solution, she would have needed only to load a software package, enter the equation, and interpret the screen display.

At a more advanced level (see fig. 2), to find the solution to the equation $2^x = 2x - 1n(x)$, a student could follow the same procedure of graphing the related function $f(x) = 2^x - 2x + 1n(x)$ and looking for the point, $x \doteq 1$, where the graph crosses the *x*-axis. In this case, because there is no elementary algebraic method of determining the solution, the use of such software opens new doors for secondary students in the study of functions and their behavior, not to speak of significant applications of mathematics.

While the foregoing examples illustrate changes at the secondary level, there are changes in the elementary- and middle-level content as well. These include a much heavier emphasis on developing students' number sense and mental mathematics skills. The ability to depict differing situations in a variety of numerical ways involves meanings associated with numbers and the abilities to represent numerical quantities with manipulatives, to visualize the relative magnitude of numbers, to discern the relative effect of operations on numbers, and to associate numbers with objects (measures) in the child's environment.

For example, when asked which two of the numbers *170*, *240*, and *420* have a sum near *400*, the child should be able to quickly identify the first two numbers, as the last one is clearly too large. When a child attempts to estimate the type of operation used on two whole numbers to obtain a given result, this too is an example of number sense. The child may reason that since the result is larger than either of the two starting numbers, the operation used must have been either addition or multiplication. Subsequent analysis of the relative magnitude of the numbers could further pinpoint the answer. These same number sense skills have related steps tied to the development of decimal and rational number concepts in the middle grades.

Teachers will need time to assess the changes that have occurred, such as the development of the calculator and computer skills across the grade levels, the rethinking of the curriculum to redefine expectations for pa-

Examples of "What if . . ." Mathematics Questions

1. How many boxes of Wheaties would one have to buy, on average, to feel confident that one would have one of each of the four different Michael Jordan posters enclosed as a premium?

[This question involves students in a probabilistic situation where the question is best illustrated by modeling the problem with a deck of cards consisting solely of an equal number of 1s, 2s, 3s, and 4s. Students would then have to find out, on average, how many cards needed to be turned over until they had at least one of each of the four numerals showing.]

2. What is the route of least distance for air travel from Honolulu, Hawaii, to Washington, D.C.? How many states will the path cross? Will it cross any other countries?

[To solve this problem, students need to integrate geometric knowledge with geographic knowledge.]

3. What is the average weight of the students in your classroom? How much weight would each individual have to gain in order to increase the average weight by five pounds? Do you need to know the current average to answer this last question?

[This problem provides an opportunity for students to see that some questions are not really computational items at all but, rather, require only an understanding of the underlying concepts.]

4. Can you slice a cube in order to get a cross section that is a square, a non-square rectangle, a pentagon, a hexagon, a septagon, or an octagon?

[This problem allows for experimentation and opportunities for a realm of reasoning for possible answers.]

5. Given a road map of greater Los Angeles, what are reasonable routes to drive from Fullerton to Pasadena? Which route appears to be the shortest? How did you determine your answer? Are there times of the day when other routes might be more attractive? If so, why?

[This question allows for the development of a mathematical solution and then the consideration of the practicality of that solution when it is tempered by other extenuating circumstances.]

per-and-pencil skills in terms of conceptual expectations tied to number sense and estimation, and the opening of the curriculum to those who have traditionally been filtered out by failure on rote drill exercises. Teachers will require time, in view of these developments, to carefully coordinate their programs, study new curricular materials, and develop supporting materials for their classrooms.

The Direction Ahead

The recommendations set forth in the *Standards* will offer assistance to districts, schools, and teachers as they make significant changes in the mathematics curriculum over the next decade. Successful school programs in mathematics will have to attend to a number of points:

Recommendation 1: Schools should make a zero-based analysis of the content of their mathematics curriculums. Time for mathematics is valuable and already too crowded with content. Each topic allotted time in the curriculum must be justified on the basis of the role it plays in the students' overall mathematical growth.

Recommendation 2: Calculators and computers must be integrated into the curriculum as "fast pencils." Their primary use must be to support the overall growth of concepts and processes, not just to quickly check answers or serve as objects to teach programming skills.

Recommendation 3: Recent and relevant applications of mathematics must be given attention to illustrate the value of mathematics, as well as to provide motivation for the study of mathematics.

Recommendation 4: Teachers must participate in continued professional development to keep abreast of changes in the content of the curriculum and to learn how to structure it for classroom settings.

Recommendation 5: States, schools, and teachers must work *with* publishers and test developers to develop materials that adequately present mathematics to children and assess their progress toward the goals outlined by the *Standards*.

Recommendation 6: Revisions of school mathematics programs must involve teachers, administrators, parents, mathematics educators, and others interested in the mathematical development of our nation's youth.

A Basic Right

Efforts directed at accomplishing these recommendations—be they at the state, school, or building level—will result in significant progress toward the spirit of the curriculum envisioned by the *Standards*. Accompanying these efforts must be the articulation of local change with state and national change to ameliorate problems created by differing expectations, our mobile society, and varying opportunities to learn afforded by schools. Mathematics must become, for children, a basic right afforded to all in a manner that provides each child with the power required to face mathematics situations with confidence and visions of success. □

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