Trends

Mathematics

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Three Promising Developments in Testing

For more than 15 years, leaders in mathematics education have been advocating more emphasis on higher-order thinking skills, problem solving, communication, estimation, helping people live with mathematical uncertainty, and so on. Teachers, textbooks, curriculum guides, society, and the students themselves have been criticized for not responding. Although many of us have pointed the finger at intellectually limited (and limiting) standardized tests, such tests have, if anything, become worse and more omnipresent than ever.

Children's Television Workshop (CTW) has developed some very sophisticated, open-ended, free response activities to evaluate children's use of good problem-solving behaviors and their mathematical sophistication in solving problems. The correlations between scores on regular standardized tests and these sophisticated instruments were all essentially zero (they ranged from -.18 to +.11). Examination of the CTW instruments as compared to standardized tests would convince any knowledgeable person that the instruments are much more closely aligned with recent recommendations than are standardized tests. The lack of correlation is particularly surprising since most of us have assumed standardized tests measured something that was necessary, though obviously not sufficient, for intelligent mathematical behavior. (The CTW study showed, incidentally, that watching six weeks of Square One TV increases the number and variety of problem-solving techniques and the sophistication of solutions to non-routine problems.) If teachers are teaching to standardized tests and textbook writers are writing to those tests, and if the tests fail to measure what we claim we want children to learn, how can we possibly hope children will learn these higher-order thinking skills?[2]

For some years, I have advocated either eliminating all standardized testing or using only those tests that emphasize higher-order thinking skills. Until recently, the two recommendations were equivalent. However, two interesting new developments appear to be changing this bleak picture. First, the Educational Testing Service is creating a "new generation of assessments for the purpose of licensing teachers" and is presently circulating a task analysis inventory to a sample of teachers to help develop the new assessments. From preliminary information, it appears that the new assessments will include substantial mathematical reasoning and modeling, some open-ended free response items, and even a final stage of evaluation involving direct observation and data collection from the beginning teachers' school. If states and localities choose to use the more sophisticated and appropriate parts of this successor instrument, we can expect a substantial improvement in the selection and preparation of mathematics teachers.

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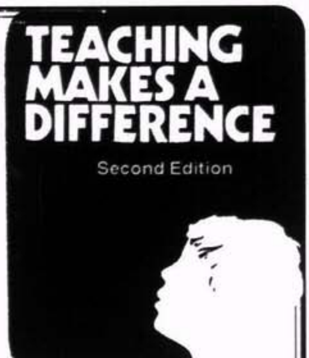
Meanwhile, California has recently begun to publish and use state assessment instruments with open-ended questions that require independent thought and well-written responses and that do not necessarily have one "best" answer. In *A Question of Thinking*,[1] a distinguished panel of California mathematics educators reviews questions and responses from the first 12th grade version of this new test. For each of five questions, they provide an explanation of what was expected, discuss the strengths and weaknesses in students' responses, and examine misconceptions and teaching implications. This is an unabashed attempt to use examinations to improve the teaching of mathematics. It is the most promising such attempt I've seen so far.

The five items discussed in *A Question of Thinking* are:

1. Imagine you are talking to a student in your class on the telephone and want the student to draw some figures. The student cannot see the figures. Write a set of directions so that the student can draw the figures exactly as shown below. [*Two figures are drawn on graph paper.*]

2. Look at these plane figures, some of which are not drawn to scale. Investigate what might be wrong (if anything) with the given information. Briefly write your findings and justify your ideas on the basis of geometric principles. [*Three figures are given, a triangle in which the sum of the angles is not 180°, an unlikely looking trapezoid that is not to scale but has no conflicting information, and a circle that seems to have a chord of greater length than its diameter, though there could be some dispute about that. The examiners, incidently, clearly look with great favor on students who think of something unexpected.*]



3. James knows that half the students from his school are accepted at the public university nearby. Also, half are accepted at the local private college. James thinks that this adds up to 100 percent, so he will surely be accepted at one or the other institution. Explain why James may be wrong. If possible, use a diagram in your explanation.

4. Consider the following problems:

a. Marla has a job after school. Last week she worked 2 hours and earned $10.50. How much did she earn per hour?

b. This week Marla worked 2 hours and earned $10.50 per hour. How much did she earn this week?

c. Marla worked two jobs. She earned $5.25 on the first and $10.50 on the second. How much did she earn all together?

- Which two problems are most similar and why?
- Which two problems are most dissimilar and why?

[*There is no best answer for this, but students are expected to choose criteria and strategies that have mathematical significance and provide clear mathematically correct discussions.*]

5. The square shown below has sides of length 2 units. Connect the midpoints of the sides of the square, in order, to form an *interior* square. Repeat the same process to make squares within squares. [*The square is drawn on quarter unit graph paper.*]

a. Draw the first five interior squares.

b. Write the sequence of numbers that represent the areas of the first five *interior* squares.

c. What rule can be used to find the nth *interior* square?

These initiatives by California, ETS, and CTW are some of the most promising recent developments in the continuing attempt to improve the teaching of mathematics. May they succeed!☐

[1]Published in 1989 and available from: Bureau of Publications, Sales Unit, California Department of Education, P.O. Box 271, Sacramento, CA 95802-0271, for $6 (plus sales tax for California residents).

[2]For more information, contact Eve Hall, Square One TV Research Department, CTW, 1 Lincoln Plaza, New York, NY 10023.

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