What Fifth Graders Learn When They Write Their Own Math Problems

An alternative approach to problem-solving instruction, elementary students in the Mathematician's Chair make connections between school math and everyday experience.

KEN WINOGRAD

American students consistently compare poorly with their international counterparts on assessments of math knowledge and problem-solving abilities (Stevenson et al. 1986). While common sense does not always lead to common practice, I believe that many of the problems associated with mathematics education could be solved with some everyday commonsense solutions.

In September 1989, Donna Strohauer, a 5th grade teacher, and I developed an alternative approach to mathematics problem-solving instruction. We had students write and solve their own original math story problems and then share these problems with peers in small cooperative groups.

The math problem-writing activity is essentially an adaptation of recent child-centered and holistic approaches to writing instruction (Graves 1983, Calkins 1986). Graves (1983), for example, encouraged students to select and develop topics for writing that reflected their personal experiences and interests. As students engaged in meaningful writing activities, they learned formal language arts skills. As an extension of Graves' approach to teaching writing, I examined the possibilities of having students reflect upon the mathematics in their everyday lives, write about this experience, and then raise related questions.

While many teachers use student-generated story problems as an enrichment experience after a page of textbook problems, we conceptualized these exercises as a regular and core component of the mathematics curriculum.[1]

Writing Challenging Problems

Students tended to write story problems that they themselves had difficulty solving or understanding, and these problems fell into one of four categories:

Problems containing new math concepts. When Nikki wrote her problem on calculating the volume of her recorder, she had some intuitive understanding of the concept of volume; however, this knowledge was insufficient for her to make sense of the problem, let alone solve it (see box).

From an instructional perspective, Nikki's problem provided an opportunity to teach her (and her peers who were ready for these ideas) some of the following: (a) the meaning of volume, (b) alternative ways to find volumes of rectangles and spheres, and (c) revision as a writing strategy (since the problem as written is slightly ambiguous).

Problems that require knowledge of a particular math procedure. Jeff wrote a problem about the sizes of various bears and how their sizes would translate into pounds of meat. The problem anticipated knowledge of the formal procedure for multiplying a whole number and mixed number (see box, p. 65). However, he had not yet learned this procedure, so he had trouble solving the problem.

Nikki's Problem

I have a recorder that is 19 inches long. The hole that goes straight through it is a long rectangle. It is 1/2 cm. each side. What is the volume of that whole?

Problems that require problem-solving knowledge the student does not yet possess. Beth was initially confused by her problem with mileage, but when shown how to make a table to organize its information, she was able to make sense of the problem and, later, approach similar problems more sensibly.

Problems the students understand but make minor errors in solving. The fourth category of difficulty that children encountered was of a metacognitive nature. They understood the problem and how to solve it. However, during solution they would make a minor error, perhaps in calculation, and then fail to evaluate the solution activity.

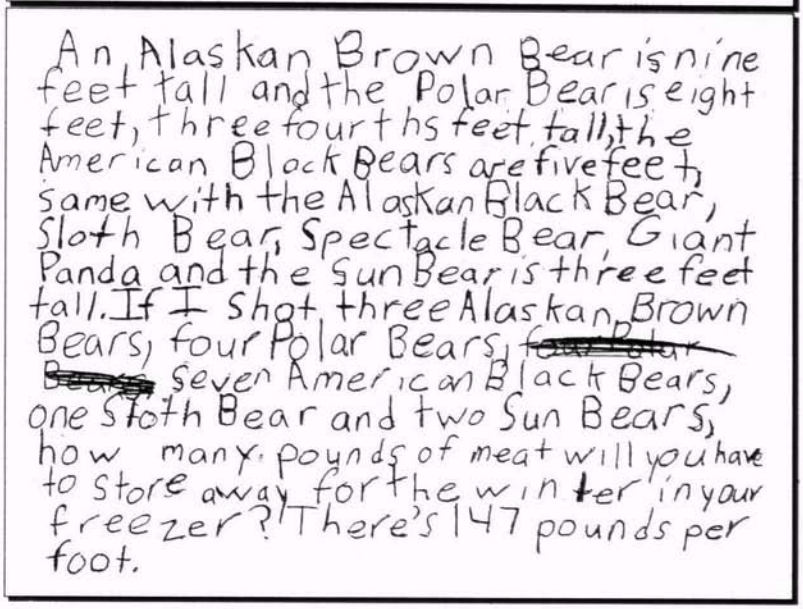
Problem Writers

Students were resourceful in the topics they generated for problems. Sources for problem topics included actual experience, hobbies, imagination, social studies or science reading, and objects in the class environment. For example, Kathleen wrote a problem about counting money, a problem that reflected an actual experience.

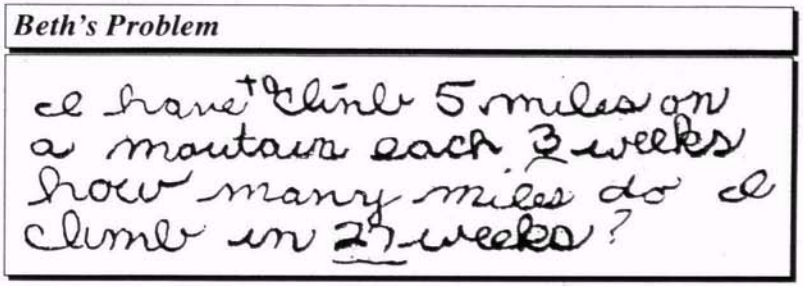
Problem writers also generated strategies to satisfy the central requirement of a story problem, which is that the story actually be problematic or difficult. Some of these strategies included (a) adding extra numerical information, (b) adding extra non-numerical information, (c) adding pertinent (to the problem) information, (d) using large or perceived difficult numbers (odd numbers were considered by children to be more difficult than even numbers), (e) avoiding a standard problem question, and (f) making a subprocedure of the problem a potential problem itself. Jessica's problem is an example of this last strategy (see box p. 66).

After writing this problem, Jessica admitted that she initially was going to specify the exact length of the school day. Instead, she decided to manipulate the information so that her peers would have to figure out the length of the school day for themselves. (Incidentally, my observations suggested that problem writers, by purposefully increasing problem difficulty for peers, often inadvertently increased problem difficulty for themselves as well.)

Jeff's Problem



Beth's Problem

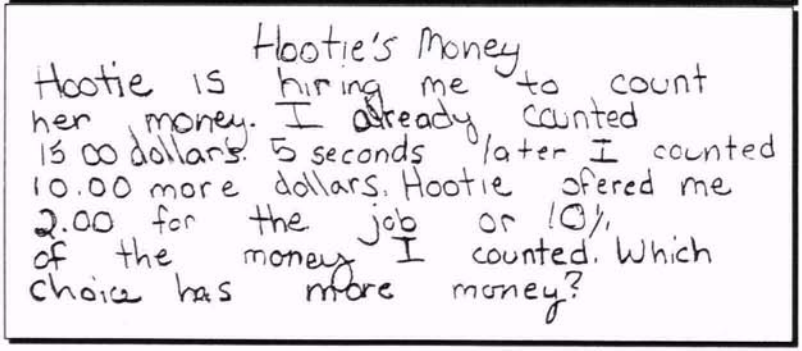


Mathematical Beliefs

According to students, the "good" math student was someone who wrote interesting and challenging problems and then worked diligently at understanding and solving those problems. Students believed that the "good" story problem was challenging, included interesting content from everyday life, and contained non-routine characteristics, such as extra information.

Students identified peer-generated problems as more interesting and challenging than textbook problems. Text-

Kathleen's Problem



book problems were easier than peers' problems, claimed students, because textbook problems tend to be grouped in chapters or on worksheets according to a common concept, skill, or procedure. After doing one or two problems on a textbook page, for example, it was not difficult for students to figure out the required operation for the remaining problems. However, because students were given carte blanche in the content of their problems, one student-generated problem was typically different from the next in its structure or solution procedure. Each student-generated problem had to be analyzed and understood independently of the preceding or following problems (see Winograd 1991).

Small-Group Behavior

When students shared problems in small groups, they usually shared problems that were difficult for one or more of the group members. Small-group talk also tended to be very on-task. The student who was sharing his/her problem usually acted as a "taskmaster" who worked to keep the group focused on the story problem at hand.

Groups tended to be more successful in solving one another's problems when the problem writer had a teaching style that was nondirective, patient, and informal. This informal teaching style gave problem-solvers the time to engage in more exploratory activities (rereading, drawing a picture, making a table) and, in effect, to develop their own *understandings* of problems.

Groups tended to be unsuccessful when the leader provided explanations or directions too quickly. In these cases, the problem-solvers did not have the time to develop their own understandings.

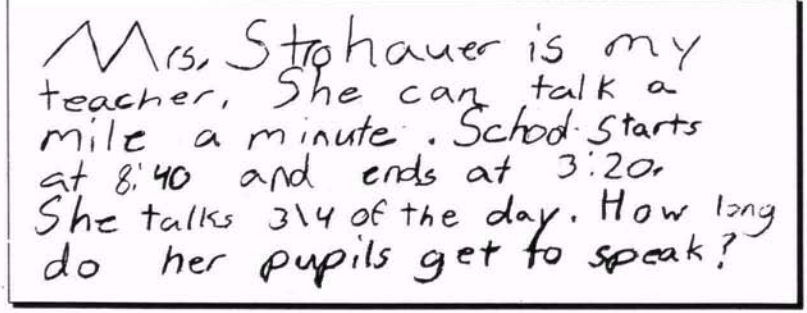
A Supportive Environment

The students were very capable and interested in initiating and sustaining their own math problem-solving activity when instruction invited them to do so. In supportive environments that invite them to ask questions, elementary students can become very fluid, energetic, and insightful problem writers.

One important rationale for having students write original math story problems is to provide a friendly context for adult instruction in math problem solving and concepts. Students tended to be more receptive to problem-solving instruction when such instruction related directly to their problems. For example, students who wanted to be more effective teachers of their problems were more attentive to problem-solving instruction when it was framed with the language, "Here are some strategies you can use when you are teaching your problem." Adult instruction that is congruent with students' immediate needs is, in the minds of students, defensible, rational, and purposeful. Therefore, regular instruction in a context of student-generated math story problems suggests very exciting possibilities for students' learning.

This study demonstrates that the kind of school math culture advocated by math educators, mathematicians, and others (see Lampert 1986, Schoenfeld 1987, Whitin et al. 1990, Albers and Alexanderson 1985, and

Jessica's Problem



Bishop 1988) is conceivable within the boundaries of the traditional elementary school. In this one 5th grade classroom, students succeeded in writing math problems on a regular basis, solving their own and peers' problems, using math-related writing as an important medium of social discourse, and making connections between school math and everyday experience and imagination.

The setting for this study was a 5th grade mathematics classroom in a middle-class, urban Colorado public school. Students wrote, solved, and shared (in small groups) original math story problems three or four days a week during the math period for the entire school year. Math periods were extended from 60 minutes to 75 minutes to accommodate both problem-writing and traditional math textbook activity (Eicholz et al. 1985).

The math period began with Mathematician's Chair, an activity in which one student shares his/her problem, usually on the chalkboard, with the entire class (an adaptation of Graves and Hansen 1983). After Mathematician's Chair (lasting about 15 minutes), students engaged in one of three activities: (a) the writing, solving, and sharing (with peers) of new story problems, (b) small-group textbook-based instruction from the teacher, and (c) an assignment from the math textbook. During the final 15 minutes of the math period, students shared their story problems in small groups (usually three students per group). One student read his/her problem to the others, who then attempted its solution. Like Mathematician's Chair, small-group sharing was highly interactive, and discussions sometimes became quite heated. For a detailed description of the problem-posing activity, see Silverman, Winograd, and Strohauer (in press) and Winograd (1990).

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