THE EVOLUTION OF CLASSROOM INSTRUCTION FOR THE COLORADO AND COMMON CORE STATE STANDARDS: Mathematics

Sandra M. Alberti,
Student Achievement Partners
Organization

- Further Q and A from plenary
- Deeper dive into the shifts and their implications
- Illustrated examples/problem sets
Mathematics: 3 shifts

1. **Focus**: Focus strongly where the standards focus

2. **Coherence**: Think across grades, and link to major topics

3. **Rigor**: Require fluency, application, and deep understanding
Shift One: **Focus** strongly where the Standards focus

- Significantly narrow the scope of content and deepen how time and energy is spent in the math classroom

- Focus deeply only on what is emphasized in the standards, so that students gain strong foundations
Focus in International Comparisons

- Move away from ‘mile wide, inch deep’ curricula identify in TIMSS
- On average, the U.S. curriculum omits only 17 percent of the TIMSS grade 4 topics compared with an average omission rate of 40 percent for the 11 comparison countries.
- The United States covers all but 2 percent of the TIMSS topics through grade 8 compared with a 25 percent non-coverage rate in the other countries.

– Ginsburg et al., 2005
Focus in International Comparisons

- High-scoring Hong Kong’s curriculum omits 48 percent of the TIMSS items through grade 4, and 18 percent through grade 8.

- “Less topic coverage can be associated with higher scores on those topics covered because students have more time to master the content that is taught.”

— Ginsburg et al., 2005
The shape of math in A+ countries

Mathematics topics intended at each grade by at least two-thirds of A+ countries

Mathematics topics intended at each grade by at least two-thirds of 21 U.S. states

### Higher Demands for Similar Content...

<table>
<thead>
<tr>
<th>Topic: Number—Numerical Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Massachusetts</strong></td>
</tr>
<tr>
<td><strong>Item</strong></td>
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<tr>
<td><strong>Item Format</strong></td>
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<tr>
<td><strong>Computational Difficulty</strong></td>
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<tr>
<td><strong>Cognitive Complexity</strong></td>
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<tr>
<td><strong>Comments</strong></td>
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</tbody>
</table>

NOTE: This item is from an alternate form of the test and is not included in Exhibits 3 or 6.

| **Massachusetts** | **Hong Kong** |
| **Item Format** | Short closed constructed-response |
| **Computational Difficulty** | High (three-digit by one-digit long division) |
| **Cognitive Complexity** | Level 3 (requires an in-depth understanding of the division algorithm and significant reasoning skills (6 × a = 36, 82 and a × 6 = 5 _). |
| **Comments** | Solution: 17 × 6 = 102, 27 × 6 = 162, 37 × 6 = 222, 47 × 6 = 282, but 4 × 6 cannot be 5 __. However, 97 × 6 is 582 and 9 × 6 = 54. Alternatively, students might start with the lower box having to be a 4 since there is no remainder and then work backward to fill in the numbers |
But Much Sharper Focus

### Exhibit 13. Number: Estimation

<table>
<thead>
<tr>
<th>Massachusetts</th>
<th>Hong Kong</th>
</tr>
</thead>
</table>
| Brianna bought 4 shirts. Each shirt cost $8.95.  
Which estimate is closest to the total cost of the shirts that Brianna bought? | The Hong Kong test includes no estimation items. |
| a. $32  b. $36  c. $38  d. $40 | |

### Exhibit 34. Patterns, Relations and Algebra—Patterns

<table>
<thead>
<tr>
<th>Massachusetts</th>
<th>Hong Kong</th>
</tr>
</thead>
</table>
| Ms. Mackey wrote the number pattern below using the rule “subtract 8.”  
187, 179, 171, ____, 165, 147, 139  
What is the missing number in Ms. Mackey's pattern? | The Hong Kong test includes no algebra pattern items. |
| a. 103  b. 168  c. 170  d. 177 | |

<table>
<thead>
<tr>
<th>Massachusetts</th>
<th>Hong Kong</th>
</tr>
</thead>
</table>
| Zoey is using bananas and oranges to make the pattern shown below. The rule for her pattern is ABBB.  
Zoey will follow the rule for her pattern a total of 4 times.  
How many oranges will Zoey use in all? Show or explain how you got your answer? | The Hong Kong test includes no algebra pattern items. |
| | |

### Exhibit 35. Patterns, Relations and Algebra—Number Sentences

<table>
<thead>
<tr>
<th>Massachusetts</th>
<th>Hong Kong</th>
</tr>
</thead>
</table>
| Which number sentence is true?  
[A] $5 + 0 = 5 \times 1$  
[B] $5 + 1 = 5 \times 1$  
[C] $5 + 0 = 5 \times 0$  
[D] $5 + 1 = 5 \times 0$ | The Hong Kong test includes no algebra number sentence items. |

<table>
<thead>
<tr>
<th>Massachusetts</th>
<th>Hong Kong</th>
</tr>
</thead>
</table>
| Candace wrote the number sentence below.  
$15 - 3 = \square$  
Which of these is another way to write Candace’s number sentence? | |
| [A] $15 + \square = 3$  
[B] $15 \times \square = 3$  
[C] $3 + \square = 15$  
[D] $3 \times \square = 15$ | |
# Traditional U.S. Approach

<table>
<thead>
<tr>
<th>K</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and Operations</td>
<td>▶️</td>
</tr>
<tr>
<td>Measurement and Geometry</td>
<td>▶️</td>
</tr>
<tr>
<td>Algebra and Functions</td>
<td>▶️</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>▶️</td>
</tr>
</tbody>
</table>
Focusing attention within Number and Operations

Operations and Algebraic Thinking

Number and Operations—Base Ten

Number and Operations—Fractions

Expressions and Equations

The Number System

Algebra

K 1 2 3 4 5 6 7 8

High School
Shift Two: **Coherence**

Think across grades, and link to major topics within grades

- Carefully connect the learning within and across grades so that students can build new understanding onto foundations built in previous years.

- Begin to count on solid conceptual understanding of core content and build on it. Each standard is not a new event, but an extension of previous learning.
Coherence example: Progression across grades

“The **coherence** and sequential nature of mathematics dictate the foundational skills that are necessary for the learning of algebra. The most important foundational skill not presently developed appears to be proficiency with fractions (including decimals, percents, and negative fractions). **The teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected.**”

<table>
<thead>
<tr>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 6</th>
</tr>
</thead>
</table>
| Addition and subtraction:  
  - Add and subtract like fractions  
  - Add and subtract related fractions (denominators of given fractions should not exceed 12)  
  - Multiplication and division of fractions:  
    - Multiply proper fractions, improper fractions, mixed numbers and whole numbers by proper fractions, improper fractions, and mixed numbers  
    - Divide fractions by whole numbers and whole numbers by fractions | Addition and subtraction of fractions with unlike denominators:  
  - Add and subtract fractions with unlike denominators  
  - Division of fractions:  
    - Divide proper fractions by proper fractions  
    - Mixed calculations with fraction and decimal:  
      - Know how to solve simple calculation with both fractions and decimals | Division of fractions:  
  - Divide proper fractions by proper fractions |


**CCSS**

<table>
<thead>
<tr>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</td>
<td>5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</td>
<td>6.NS. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</td>
</tr>
<tr>
<td>5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.</td>
<td>6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.</td>
<td></td>
</tr>
</tbody>
</table>
**Coherence example: Grade 3**

The standards make explicit connections at a single grade

- **Multiplication and Division**: 3.OA.5
- **Properties of Operations**: 3.MD.7c
- **Area**: 3.MD.7a
Shift Three: **Rigor**

**Equal intensity in conceptual understanding, procedural skill/fluency, and application**

- The CCSSM require a balance of:
  - Solid conceptual understanding
  - Procedural skill and fluency
  - Application of skills in problem solving situations

- This requires equal intensity in time, activities, and resources in pursuit of all three
(a) Solid Conceptual Understanding

- Teach more than “how to get the answer” and instead support students’ ability to access concepts from a number of perspectives.

- Students are able to see math as more than a set of mnemonics or discrete procedures.

- Conceptual understanding supports the other aspects of rigor (fluency and application).
(b) Fluency

- The standards require speed and accuracy in calculation.

- Teachers structure class time and/or homework time for students to practice core functions such as single-digit multiplication so that they are more able to understand and manipulate more complex concepts.
**Finger Fun for Facts with 9**

**Work Together • Visual Thinking**

You can use your fingers to calculate any fact that has 9 as a factor.

**Step 1**  Hold your hands up with palms facing you.

**Step 2**  Mentally number your fingers 1–10 as shown. These numbers will stand for the factor that is not 9.

**Step 3**  To find $6 \times 9$, bend the “6” finger (your right pinky) down.

**Step 4**  The fingers to the left of the pinky are tens. The fingers to the right of the pinky are ones.

There are 5 tens and 4 ones.

So, $6 \times 9 = 54$.

---

Write the fact that is shown by the fingers.

1. $9 \times 5 = 45$

2. $9 \times 2 = 18$

---

Sit beside a partner. One person chooses any fact below and shows it with fingers. The other person finds that fact and writes the product. Take turns until you complete all the facts.

- $9 \times 3 = 27$
- $4 \times 9 = 36$
- $7 \times 9 = 63$
- $9 \times 9 = 81$
- $9 \times 2 = 18$
- $6 \times 9 = 54$
- $9 \times 5 = 45$
- $9 \times 8 = 72$
- $9 \times 1 = 9$
# Required Fluencies in K-6

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluency</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
</tr>
<tr>
<td>2</td>
<td>2.OA.2</td>
<td>Add/subtract within 20 (know single-digit sums from memory)</td>
</tr>
<tr>
<td></td>
<td>2.NBT.5</td>
<td>Add/subtract within 100</td>
</tr>
<tr>
<td>3</td>
<td>3.OA.7</td>
<td>Multiply/divide within 100 (know single-digit products from memory)</td>
</tr>
<tr>
<td></td>
<td>3.NBT.2</td>
<td>Add/subtract within 1000</td>
</tr>
<tr>
<td>4</td>
<td>4.NBT.4</td>
<td>Add/subtract within 1,000,000</td>
</tr>
<tr>
<td>5</td>
<td>5.NBT.5</td>
<td>Multi-digit multiplication</td>
</tr>
<tr>
<td>6</td>
<td>6.NS.2,3</td>
<td>Multi-digit division</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Multi-digit decimal operations</td>
</tr>
</tbody>
</table>
Fluency in high school

**Fluency Recommendations**

A/G  
Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

A-APR.1  
Fluency in adding, subtracting and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

A-SSE.1b  
Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square and other mindful algebraic calculations.

PARCC Model Content Frameworks for Mathematics  
October 2011
(c) Application

- Students can use appropriate concepts and procedures for application even when not prompted to do so.

- Provide opportunities at all grade levels for students to apply math concepts in “real world” situations, recognizing this means different things in K-5, 6-8, and HS.

- Teachers in content areas outside of math, particularly science, ensure that students are using grade-level-appropriate math to make meaning of and access science content.
It starts with **Focus**

- The current U.S. curriculum is ‘a mile wide and an inch deep.’

- **Focus** is necessary in order to achieve the rigor set forth in the standards

- Remember Hong Kong example: more in-depth mastery of a smaller set of things pays off
## Content Emphases by Cluster

### Grade Four

<table>
<thead>
<tr>
<th>Key:</th>
<th>Major Clusters;</th>
<th>Additional Clusters;</th>
<th>Supporting Clusters</th>
</tr>
</thead>
<tbody>
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</tbody>
</table>

### Operations and Algebraic Thinking
- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

### Number and Operations in Base Ten
- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

### Number and Operations--Fractions
- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

### Measurement and Data
- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data
- Geometric measurement: understand concepts of angle and measure angles.

### Geometry
- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
Hundreds, Tens and Ones

a. 234 = _____ hundreds, _____ tens, _____ ones

b. 809 = _____ hundreds, _____ tens, _____ ones

c. 571 = _____ hundreds, _____ tens, _____ ones

d. 160 = _____ hundreds, _____ tens, _____ ones

e. 67 = _____ hundreds, _____ tens, _____ ones

f. _________ = 3 hundreds, 4 tens, 8 ones

g. _________ = 6 hundreds, 0 tens, 2 ones

h. _________ = 0 hundreds, 0 tens, 5 ones

i. _________ = 0 hundreds, 7 tens, 0 ones

j. _________ = 9 hundreds, 9 tens, 9 ones
The chart below shows the numbers of visitors at four parks last year.

<table>
<thead>
<tr>
<th>Park</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>River Park</td>
<td>8346</td>
</tr>
<tr>
<td>Oak Park</td>
<td>9103</td>
</tr>
<tr>
<td>Lake Park</td>
<td>9088</td>
</tr>
<tr>
<td>Green Park</td>
<td>6299</td>
</tr>
</tbody>
</table>

Which park had the most visitors last year?

- (A) River Park
- (B) Oak Park
- (C) Lake Park
- (D) Green Park
Make true equations. Write one number in every space. Draw a picture if it helps.

1) 1 hundred + 4 tens = ______

2) 4 tens + 1 hundred = ______

3) 14 tens = __10__ tens + _____ tens
   = _____ hundred + __4__ tens
   = ______

4) 7 ones + 5 hundreds = ______

5) 8 hundreds = ______

6) 106 = __1__ hundred + ______tens + ______ones

7) 106 = ______tens + ______ones

8) 106 = ______ones

9) 90 + 300 + 4 = ______

Are these comparisons true or false?

10) 2 hundreds + 3 ones > 5 tens + 9 ones

11) 9 tens + 2 hundreds + 4 ones < 924

12) 456 < 5 hundreds
Fractions

- Draw a picture that shows why \( \frac{14}{6} = \frac{7}{3} \). Use a rectangle or a circle for the unit whole.

- Use the number line diagram to show why \( \frac{14}{6} = \frac{7}{3} \). Show the unit whole.
Write four fractions that all equal 7:
____, ____, _____, ______.

Write a fraction that equals 7 and has denominator 7: ______.

Which number is least and which is greatest?
3/4, 2, 4/4, 3/5
The gas station is 3/4 mile from my house. The repair shop is four times farther away. How far is the repair shop from my house?

To make a time-lapse video, a camera shutter was opened 125 times. Each time, the shutter was open for 3/4 of a second. How long in total was the camera shutter open? Between what two whole numbers does your answer lie?
Algebra

Write an equation with the same solutions as $3(y - 1) = 8$: ________________

If $3 - x + x - x + x - x + x - x + x - x = 0$, then what is $x$? _______

How many real numbers satisfy the equation $x^2 = x$? ________________

Write $6^3 + 7 \times 4$ as a whole number: _______
Resources

- www.illustrativemathematics.org
- http://ime.math.arizona.edu/progressions/

Products

- Draft 3–5 progression on Number and Operations—Fractions
- Data part of the K–5 progression on Measurement and Data
- Draft K–5 Progression on Number and Operations in Base Ten
- Draft K–5 Progression on Counting and Cardinality and Operations and Algebraic Thinking (Now includes all of K–5 OA)
- Draft 6–8 Progression on Expressions and Equations
- Draft 6–7 Progression on Ratios and Proportional Relationships