Students learning math are expected to do more than just solve problems; they must also be able to demonstrate their thinking and share their ideas, both orally and in writing. As many classroom teachers have discovered, these can be challenging tasks for students. The good news is, mathematical communication can be taught and mastered.

In *Teaching Students to Communicate Mathematically*, Laney Sammons provides practical assistance for K–8 classroom teachers. Drawing on her vast knowledge and experience as a classroom teacher, she covers the basics of effective mathematical communication and offers specific strategies for teaching students how to speak and write about math. Sammons also presents useful suggestions for helping students incorporate correct vocabulary and appropriate representations when presenting their mathematical ideas.

This must-have resource will help you help your students improve their understanding of and their skill and confidence in mathematical communication.

Laney Sammons, a former classroom teacher and instructional coach, is an educational consultant. She has worked with teachers of all grade levels throughout the United States and Canada to help them improve their mathematics instruction.
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Communication is an essential part of mathematics and mathematics education. It is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process also helps build meaning and permanence for ideas and makes them public (National Council of Teachers of Mathematics [NCTM], 2000, p. 60).

In mathematics, as in all subject areas, the true essence of teaching is guiding others to greater understanding. Exemplary math teachers nurture their students’ appreciation of the discipline and lead them to an understanding of math that can be applied in diverse situations. This kind of teaching does more than simply impart facts and procedures that are devoid of context or meaning; it taps into the curiosity of learners and offers them opportunities for mathematical exploration, with teachers and learners working collaboratively to construct knowledge (Mercer, 1995). Essential to this learning process is effective communication.

What Is Mathematical Communication—and Why Write a Book About It?

Merriam-Webster (2017) defines communication as “a process by which information is exchanged between individuals through a common system of symbols, signs, or behavior.” As such, mathematical communication entails
a wide range of cognitive skills. Because it is an exchange of ideas, it encompasses both listening and reading (comprehension) and both speaking and writing (expression). Somewhat unique to math, expression may also include the representation of mathematical ideas in nonlinguistic ways.

Whereas math teachers have traditionally focused on teaching content, more challenging standards are encouraging educators to expand their instruction to promote students’ mathematical practice skills, most of which depend heavily on learning to communicate effectively about math. As a later summary of current math practice standards will show, most educators agree that effective communication is critical for more rigorous instruction and deeper mathematics learning, and many teaching materials and resources now include tasks that require it. Too often, however, students receive little or no instruction on how to communicate about math effectively before they are asked to do so. Naturally, this sets them up for failure. Just requiring students to justify their reasoning does not work; they must know what makes math talk and math writing effective. Students should be explicitly taught these essential skills and given ample opportunities to practice independently, beginning in kindergarten.

This book is designed to provide educators with strategies for teaching students to express their mathematical thinking effectively—orally, with the use of representations, and in writing. Successful literacy strategies, including word walls, modeling, shared writing and revision, and exemplars—strategies that show students how to talk about and write about math, rather than only assigning tasks that require it without having first taught students how to do it—will be closely examined.

After establishing what mathematical communication is and why it is essential, Chapters 2–7 examine the individual components of this type of communication. In addition to the components that are most commonly the focus of teaching mathematical communication—math vocabulary, discourse, and writing—representation as a method of communicating mathematical thinking is highlighted. Sample lessons and classroom scenarios for grade level bands K–2 and 3–5 and upper grades are included throughout, with specific instructional ideas you can use with your students.
As will become clear, the benefits of engaging students in mathematical communication go far beyond helping students meet required standards or achieve higher grades. Simply by going through the process of reflecting, organizing their thoughts, and deciding how to express those thoughts in words, students learn to think more deeply, assess their own understanding, make connections, determine importance, and compare ideas. The ongoing interaction with mathematical vocabulary helps reinforce students’ understanding, not only of the words themselves, but also of the mathematical ideas the words express. The teaching ideas and examples in this book are offered as a path to more rigorous instruction in which students are immersed, with the help of effective communication, in the fascinating and challenging discipline of mathematics.

**Mathematical Communication and the Standards**

Most mathematics standards now address content *and* process. While the mathematical content for grade levels varies and builds from year to year, the processes or practices remain more consistent. They specify the ways students should learn to interact with math—how students learn to act as true mathematicians. In examining the processes and practices prescribed in these standards, the importance of teaching students how to communicate mathematically is clear. It’s worth taking a brief look at how communication is treated in the documents that currently guide instructional goals and curriculum development for K–12 mathematical instruction: the National Council of Teachers of Mathematics (NCTM) Process Standards (2000), the Common Core State Standards for Mathematical Practice (2010), and selected state standards that are largely based on the first two.

**National Council of Teachers of Mathematics Process Standards**

In 2000, the NCTM introduced a set of principles and standards for mathematics instruction that include both content and process standards. While proficiency in mathematical communication is implicit in many of the content standards, it is inherent in the NCTM process standards described here.
Problem solving. Students should be engaged in solving problems posed in math class, as well as those that occur in real-life situations. They should be encouraged to construct new mathematical meaning from their problem-solving efforts. Being able to communicate mathematically is essential for these tasks. First, students must make sense of problems, make connections to the math they know, and then translate the problems into mathematical terms. According to the NCTM (2000), good problem solvers “monitor and reflect on the process of mathematical problem solving” (p. 52) and adjust their use of strategies as needed. “Such reflective skills are much more likely to develop in a classroom environment that supports them” (p. 54). This standard requires that teachers establish a learning environment in which students develop the habit of reflection through conversation, beginning in the early grades.

Reasoning and proof. Students should understand that reasoning and proof are fundamental to the discipline of mathematics. As learners “make and investigate conjectures” or “develop and evaluate mathematical arguments and proofs” (p. 56), strong communication skills are essential.

Communication. This standard is explicit in emphasizing the importance of students being able to “organize and consolidate their thinking through communication,” as well as being able to “communicate their mathematical thinking coherently and clearly to their peers, teachers, and others” (p. 60). They must also “analyze and evaluate the mathematical thinking and strategies of others” and “use the language of mathematics to express mathematical ideas precisely” (p. 60).

Connections. Learners should “recognize and use connections among mathematical ideas” and “understand how mathematical ideas interconnect and build upon one another to produce a coherent whole” (p. 64). They should also be able to “recognize and apply mathematics in context outside of mathematics” (p. 64). Communicating their ideas is valuable in leading students to clarify and organize their thinking more effectively and to help them recognize important mathematical connections.

Representation. This standard specifically states that students create and use representations to “organize, record, and communicate
mathematical ideas” (p. 67), underscoring the crucial role of communication in mathematical proficiency.

**Common Core State Math Standards of Mathematical Practice**

The Common Core State Standards include eight Standards for Mathematical Practice that apply to students from kindergarten to 12th grade. Briefly, students should be able to perform the following important tasks:

- Make sense of problems,
- Reason abstractly,
- Construct arguments and critique the reasoning of others,
- Construct mathematical models,
- Use appropriate tools,
- Attend to precision,
- Make use of structure, and
- Look for and express regularity in repeated reasoning.

In examining these tasks, it is obvious that communication is key to many of them. To construct mathematical models, students must construct representations of mathematical thinking—a crucial element of communication. As well, to construct arguments, critique the reasoning of others, attend to precision, or express regularity in repeated reasoning, students must be able to clearly communicate their mathematical thinking.

**Standards in Various States**

Various states have adopted their own math standards that address specific processes or practices. For example, the Texas Essential Knowledge and Skills for Mathematics (2012) highlight the process standards and their important relationship to mathematical content instruction with this explanation:

The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and
use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course.

In some states, while there may be no specific process or practice standards, they are still considered essential for mathematical proficiency. The Mathematics Standards of Learning for Virginia Public Schools, for example, establish five goals for mathematics instruction:

- Becoming mathematical problem solvers,
- Communicating mathematically,
- Reasoning mathematically,
- Making mathematical connections, and
- Using mathematical representations to model and interpret practical situations (Board of Education of the Commonwealth of Virginia, 2016, p. v).

The Nebraska Department of Education (2015) also describes four mathematical processes—problem solving, modeling and representation, communication, and making connections—stating that they “reflect the interaction of skills necessary for success in math coursework, as well as the ability to apply math knowledge and processes within real-world contexts” (p. 2).

The Importance of Teaching Mathematical Communication

The individuals who develop math standards recognize the importance of teaching students of all grade levels how to communicate mathematically. Incorporating ongoing opportunities for mathematical communication (oral, representational, or written) as an integral part of instruction not only enhances student learning, but also provides students with much-needed life skills. To avoid what Wagner (2008) calls a “global achievement gap,” schools are working to extend instruction beyond the simple acquisition of knowledge to reflect the demands of life in the 21st century. Educators are helping students learn how to think critically, work collaboratively with their peers, access and accurately analyze relevant information, and solve problems effectively. Because the ability to communicate is crucial for these important life skills, creating classroom environments
in which students regularly practice multiple forms of communication is imperative.

**Building Mathematical Comprehension**

As noted, the standards incorporate mathematical communication as an essential competency in and of itself—something required of mathematicians and necessary for meeting the everyday demands of life in our society. However, participating in oral and written communication also enhances students’ conceptual understanding of mathematics; by melding their own ideas with those of others, their mathematical understanding is refined and expanded.

Furthermore, thoughtful communication requires careful reasoning. As Chapin and colleagues (2003) pointed out, “We reason when we examine patterns and detect regularities, generalize relationships, make conjectures, and evaluate or construct an argument” (p. 79). “Being asked, ‘Why do you think that?’ has profound effects on students’ mathematical comprehension and on their ‘habits of mind’ in general” (p. 19).

Students preparing to share their thinking must incorporate the following activities:

- Review what they know about math related to the topic,
- Make mathematical connections,
- Organize their ideas,
- Determine the relative importance of their ideas to the math topic,
- Decide which ideas to share with others,
- Identify the appropriate mathematical vocabulary terms to use when communicating their ideas,
- Compose a statement that clearly explains their ideas, and then
- Express their thinking orally, with representations, and/or in writing.

Additionally, students must use the following skills in conversations or when reading other students’ written communications:

- Listen to or read others’ mathematical ideas,
- Compare those ideas to what they already know and think,
• Construct new knowledge or meaning by melding the new ideas with their own thinking,
• Decide what thoughts to include in a response,
• Compose a response, and
• Deliver the response.

The entire process begins anew when students listen to or read responses from other students. When that happens, students must attend to any feedback offered by peers or teachers and then cycle back through the steps of this thinking process.

With each step, students revisit and reconsider relevant mathematical ideas, often in a new light. By touching on these mathematical ideas repeatedly while communicating, students’ mathematical understanding broadens and deepens. As a result, fledgling mathematicians can apply what they are learning and are more likely to retain their new knowledge and skills.

With many opportunities to communicate their thinking, the mathematical understanding of students grows deeper and becomes more complex. Students learn “the power that comes from wrestling with an inkling of an idea, shaping and articulating it the best they can, then working with others to enable their idea to gain strength and grow” (Nichols, 2006, p. 33). This is mathematical learning at its best, going well beyond the simple acquisition of facts, procedural knowledge, and computational fluency.

**Making Mathematical Thinking Visible**

The critical role of formative assessment in both teaching and learning has been well documented (Black & Wiliam, 2010; Fisher & Frey, 2007; Stiggins, 1997, 2002, 2005). Listening to and reading about students’ explanations of their mathematical thinking offers teachers an accurate assessment of students’ knowledge and skills and helps them target specific learning needs. It is this “kind of thoughtful practice that drives effective teaching” (Rowan & Bourne, 2001, p. 37). In a sense, students begin to think as teachers: What should I know and be able to do? What are my learning goals? Have I met them? What do I have to do to meet my goals?

When teachers expect students to share and justify their mathematical reasoning, they provide rich opportunities for self-assessment. By delving
more deeply into their thinking to share it with others in mathematical conversations or writing, students often discover gaps in their understanding or lingering questions. Reflecting on their own mathematical understanding related to their learning goals enables students to assume greater responsibility for assessing their own understanding and identifying future learning goals. The extent of their understanding and their need for additional instruction become more visible to them, as well as to their teachers.

The value of this kind of cognitive activity is borne out by research. In his synthesis of over 800 meta-analyses of the influences on student achievement, Hattie (2009) identified the positive impact of visibility in both teaching and learning: “What is most important is that teaching is visible to the student and that the learning is visible to the teacher. The more the student becomes the teacher and the more the teacher becomes the learner, then the more successful are the outcomes” (p. 25). When students are actively engaged in multiple methods of communication, the roles of teacher and learner intermingle. As a result, teaching and learning become more visible to both teachers and learners.

**Teaching Students to Communicate About Mathematics**

Despite overwhelming agreement as to the value of students mastering both mathematical content and the mathematical processes or practices, classroom instruction too often offers only limited opportunities for students to acquire these skills. While many math resources now include tasks calling for students to engage with both content and process, teachers are usually given little guidance on how to teach the mathematical practices themselves. Because communication plays such a large role in these processes, strategies for teaching communication are of particular importance and are the focus of this book.

**Providing Support for Mathematics Language Learners**

For many students, learning to communicate effectively about mathematics is comparable to learning a new language. Because they encounter little content-specific mathematical vocabulary in everyday conversations, it is often unfamiliar to them. In fact, Thompson and colleagues (2008) suggest that “we should consider every student a mathematics language learner
regardless of his or her level of English language proficiency” (p. 11) and provide appropriate language acquisition support. “Just as learning a foreign language is easiest when the learner is thoroughly immersed in the language, the same principle holds true for learning mathematical vocabulary” (Sammons, 2011, p. 60) and for learning how to communicate mathematical thinking effectively.

The acquisition of new languages requires learning how to both comprehend and express ideas. The comprehension of meaning comes from reading and listening. Clearly expressing ideas, on the other hand, requires proficiency in writing and speaking (Chun, 2006). Combined, these skills “are tools for collaboration, discovery, and reflection” (Whitin & Whitin, 2000, p. 2). Effective communicators can both understand the communications of others and express their own ideas so that others understand them. In teaching students to effectively communicate mathematically, teachers need to address reading, listening, writing, and speaking (see Figure 1.1).

Because of the links between language acquisition and the development of mathematical communication skills, math teachers often look to literacy

![Skills Required to Effectively Communicate Mathematically](image-url)
instruction for effective teaching ideas. Literacy strategies are easily adapted for use in teaching math students how to express their ideas more effectively and understand the ideas communicated by others.

Mathematical communication, however, consists of considerably more than just the language components. As it is specific to mathematical content, communication is dependent upon a certain degree of mathematical background knowledge. That knowledge is the lens through which a reader or listener makes sense of the words and representations shared by others. Likewise, students need a combination of math content knowledge and the ability to express their ideas precisely when speaking or writing about math so that others can understand them.

**Establishing the Criteria for Meaningful Communication**

It is important for both teachers and students to know what constitutes quality mathematical communication. First, teachers must clarify the criteria for successful performance by students. State standards provide some guidance regarding specific expectations, but to guarantee consistency across grade levels in schools or school districts, it is beneficial for teachers to come together to examine and interpret those standards. By analyzing and assessing samples of student communication relative to the standards, teachers can develop consistent criteria for quality communication to guide both teaching and assessment.

Second, the criteria for success must be shared with students so that they have clear performance targets. Students will only improve their mathematical communication skills if they understand what defines a quality response. The following characteristics of meaningful mathematical communication should be considered when establishing criteria for quality oral and written communication:

- Respectful dialogue with others
- Use of accurate mathematical vocabulary
- Precision in the expression of ideas
- Organized and logical structure
- Use of facts to justify mathematical reasoning
• Participatory listening during conversations
• Careful reading of written mathematical communications
• Comprehension of the oral or written expressions of others
• Requests for clarification, when needed
• Responses that appropriately address the content of communications heard or read
• Disagreement expressed without rancor and supported with evidence
• Maintenance of focus (attentive, staying on topic)

Incorporating Mathematical Communication into Content Lessons

As with any of the mathematical processes or practices, mathematical communication skills are not taught in isolation. Integration of communication into content instruction is crucial to provide authentic contexts. As a part of a content lesson, communication skills may be highlighted in myriad ways:

• Teachers may explicitly teach a communication strategy combined with content in a minilesson or think-aloud.
• Participatory listening skills may be taught and practiced as students listen to their peers describe math concepts or as they brainstorm problem-solving strategies.
• Comprehension strategies may be taught to help students as they decipher word problems.
• Whole-class or small-group lessons may include math conversations focusing on aspects of the content being taught.
• Students may be asked to justify their problem-solving efforts in journals or digital presentations.
• If a class collaboratively collects and displays a set of data graphically, the teacher may lead a shared writing task to communicate the gist of what the data reveals.
• Students might be asked to brainstorm multiple ways to represent a mathematical idea or problem.

It is important to emphasize, however, that there can be no meaningful mathematical communication without mathematical content. Note that
these suggested instructional ideas all link to and rely upon the mathematical content being taught.

**Asking Questions that Develop Mathematical Communication Skills**

The chapters that follow present numerous examples of how to encourage and develop students’ communication skills in relation to specific math concepts and tasks. Although mathematical tasks in the classroom should always be linked to specific content, there are content-generic prompts, such as those that follow, that can be used to encourage students to communicate their mathematical thinking in a variety of ways and within the context of many kinds of tasks or problems:

- Reflect on your own mathematical thinking about _______. What do you understand? What questions do you have?
- Explain your mathematical thinking to another, either orally or in writing, using representations.
- Justify your own or a peer’s problem-solving process.
- Respond to the mathematical ideas of another.
- Explain a mathematical concept or problem so that others will understand it.
- Read and then restate a problem in your own words or represent it in such a way that others will understand it.
- Describe the strategies you used to solve a problem.
- Describe a pattern you notice.
- Make and justify a conjecture based on your observations.
- Share what you wonder about a problem, an observation, or a mathematical idea.
- Explain in detail your mathematical observations.
- Define mathematical terms or concepts in your own words.
- Describe how your math work might be beneficial in other situations.

These are just a few examples of student tasks that provide practice in mathematical communication. Their aim is prompting students to reason, reflect, listen, read, write, speak, represent, justify, observe, and
engage more deeply in mathematical thinking as they work to hone their communication skills.

Creating a Classroom Environment Where Mathematical Communication Flourishes

Students of all ages who regularly engage in sharing their mathematical thinking begin to assume a greater responsibility for self-regulating their thinking and their mathematical learning (Chapin, O’Connor, & Anderson, 2003). It encourages them to focus more intently on the mathematical content they are learning, on how well they are learning it, and on what they can do to improve their learning. Establishing an expectation that students express their mathematical thinking orally and in writing is crucial in creating a rich learning environment.

As stated previously, productive student communication “doesn’t just occur because the standards demand it or the teacher values it” (Sammons, 2010a, p. 40). An environment in which students productively engage in the sharing of their mathematical thinking is created with intent, offers authentic purposes for communication, and supports learners as they develop a more meaningful understanding of the world of mathematics. There are a number of things that teachers can do to make the climate of their classrooms “communication-friendly” for students:

1. Consider how the physical arrangement of the classroom impacts communication. A classroom in which students are seated in rows of desks tends to inhibit conversation. On the other hand, students who sit so that they face each other can easily engage in sharing their thinking. “Round tables, as opposed to individual desks, offer a gathering place where all come on equal terms. There is no head of the table, no one person elevated above the others” (Nichols, 2006, p. 38).

   Additionally, a carpeted gathering area where students can sit together on the floor offers an inviting environment for group conversations. By situating this meeting space adjacent to an interactive whiteboard or chart paper easel, ideas may be recorded to provide a
visual representation of the mathematical reasoning being shared or to provide future reference for the class. Making manipulatives accessible during math conversations also allows students to model their mathematical thinking to present it more clearly to others or to explore their own understanding as others share ideas.

To encourage written communication, materials such as journals, graph paper, markers, and common mathematical tools should be available to students. In addition, students can use digital devices to communicate their thinking. Slide shows, blog posts, animations, and recorded oral explanations of thinking are just a few examples of methods now available to students when they communicate mathematically. As with more traditional methods of written communication, making these easily accessible to students encourages their use.

2. Establish a nonthreatening classroom atmosphere in which errors are viewed as an important part of the learning process. O’Connell (2007) wisely warns that “there is nothing that will stifle communication more quickly than negative reactions from teachers or other students” (p. 13). Students too often believe that the sole purpose of answering a teacher’s question is to give the one and only correct answer. Students are more willing to share their thinking, however, when they understand that their mathematical thinking is highly valued.

Students’ comments that reveal misconceptions or errors guide teachers in meeting their instructional needs. If one student has a misconception, it is likely that others share it. By attending carefully to the oral and written communication of their students, teachers can dispel confusion and correct misconceptions in a timely manner. Teachers help students understand this by explicitly explaining why they consider the communication of ideas to be so valuable. When students understand that errors are an integral part of the learning process, they are much more willing to communicate their thinking openly.

3. Share expectations with students by establishing criteria for quality oral, representational, and written mathematical communication. Students never arrive at school knowing what constitutes quality mathematical
communication; they must be taught. Unless expectations are shared with students, they have little idea of the target they are aiming for. Teachers can offer students guidance regarding their expectations in many ways: modeling and thinking aloud as they communicate, discussing and posting specific criteria for success, providing exemplars of quality communication, and having students examine and analyze examples (videos or written documents) of communication to assess their effectiveness. The chances of students becoming proficient mathematical communicators greatly improve when students are aware of exactly what they are working to achieve.

4. Create a communication-rich classroom. Although the focus of this book may be mathematical communication, the more students speak, write, and share their thinking in any curricular area, the more adept they become at communicating in general. The benefits of this skill then carry over and enhance mathematical communication.

Cross-curricular communication opportunities abound in self-contained classrooms. In departmentalized math classes, however, more planning may be required. A teacher could post and highlight examples of all kinds of mathematical communication, including newspaper or magazine articles related to math, articles from mathematical blogs, graphic displays of data, mathematical conjectures, or explanations of problem-solving strategies by students. These help students, not only by providing examples they can examine and emulate, but also by making students more aware of the relevance of mathematics in the real world.

When planning lessons, include time daily for discussions of mathematical issues. Support accurate student communication by providing math word walls to reinforce mathematical vocabulary development. Engage students in shared writing activities to create anchor charts and then post them as models of mathematical writing and for future reference by students. Be attentive to examples of mathematical communication that are encountered and search for ways to incorporate them into the classroom.
5. *Provide authentic contexts for mathematical communication by students.* Students are more inclined to think deeply about math and share their thinking when the context is authentic, particularly when their curiosity is aroused about a math-related matter. Literacy author Fox (1993) suggests, “We’re currently wasting a lot of time by giving unreal writing tasks in our classrooms … . You and I don’t engage in meaningless writing exercises in real life—we’re far too busy doing the real thing” (p. 4). In both oral communication tasks and written tasks, students are most engaged when they are “doing the real thing.”

Learning is also authentic when students seek to satisfy their own curiosity. Encourage a sense of inquiry in students. Take advantage of teachable moments to spur students’ interest in and wonder about the complex discipline of mathematics. Share this wonder with learners. When students are intrigued and see purpose in their communication, their motivation increases, as does the quality of their work.

Designing and implementing an environment that encourages mathematical communication by students is just the first step in teaching students how to share their mathematical ideas effectively. The remaining chapters of this book will address the various forms of mathematical communication, both comprehensive and expressive, and offer specific instructional strategies for helping students become competent communicators. We begin with mathematical conversations—or the skills of speaking and listening—in Chapter 2.
Writing is a process through which we record our observations, our thoughts, and our insights. It is a process through which we reorganize our ideas, develop conjectures, and gain insights. It is not content in itself, but a means of exploring and expressing content. So why would we not write in math class? (O’Connell and Croskey, 2008, p. 60)

If writing about math is so beneficial, why, then, is it not more common in math classes? Teaching students how to write about math is a challenge for many educators. Just as there has been little clarity about the kinds of mathematical writing students should do, minimal guidance has been provided about how to teach it. Even when teachers know what kinds of writing their students should be doing, they must then figure out how to teach them to do it.

The traditional instructional focus in math classes has been primarily on calculation and procedural skills, with little emphasis on developing students’ conceptual understanding or their ability to communicate mathematically. This is the kind of math instruction most teachers experienced when they were in school, and many tend to teach in the same ways that they were taught. Because most were expected to do very little, if any, mathematical writing, they have scant experience upon which to draw to teach those skills. Yet, if teachers do not teach them, how are students to learn how to write mathematically?
Many elementary school teachers teach writing as well as math, but there has been very little carryover of literacy instructional strategies into math classes. Too often, there is an almost invisible barrier between subject areas that seems to segregate teaching methods to specific disciplines. Those teachers who focus only on math often have had very little training in how to teach writing and have few resources to draw upon to support this kind of instruction. Most math texts now include mathematical writing tasks of varying quality, but rarely offer teachers adequate guidance in how to develop their students’ writing skills.

**Applying Literacy Instructional Strategies to Teach Mathematical Writing**

It makes sense for math teachers to look to literacy instructional strategies for ideas in teaching mathematical writing. Mathematical writing is, after all, a subset of writing in general. With recent changes in writing standards, there is an increased focus on both informative/explanatory and argument/opinion writing. Most mathematical writing neatly fits into one of those two categories. This means, of course, that in self-contained classrooms, mathematical writing may be taught not only during math class, but also during the language arts period, if desired.

The importance of supportive writing instruction is especially clear to kindergarten teachers. While some young learners arrive in kindergarten able to print the letters of the alphabet, write their names, or perhaps spell a few simple words correctly, even those few usually know very little about the actual process of **writing** to express meaning, as opposed to mere **handwriting**. With teacher support, these young students gradually discover that they are indeed capable of communicating their ideas with illustrations and even inventive spellings. Because few students of any age have had much experience writing about math, they face the same challenges in learning how to write about math as kindergarten students do in learning how to write in general. The same kinds of instructional support that teachers give kindergarten students are also effective in teaching learners to write about math.
Lucy Calkins (1994) spearheaded writing initiatives in schools with her work on writing workshops. She urged teachers to create environments conducive to writing in which students are respected and recognized as authors from a very young age. Implicit in her approach is the realization that students can write about their ideas even when they have not yet mastered the mechanics of writing. Her seminal work moved teachers to focus on written content, even when young authors can only communicate in writing with pictures or inventive spelling. While not minimizing the importance of writing mechanics, she encouraged teachers to celebrate the ability of students to tell stories or share information through writing, even when their work has spelling, grammar, or punctuation errors. In fact, because these young authors are excited about writing, they become more receptive to learning about these conventions of writing.

Math students benefit from the same inviting approach to writing. When their mathematical ideas are respected, even very young learners are eager to share them, whether orally or in writing. Teachers have the responsibility of not only welcoming early efforts at mathematical written communication, but also providing meaningful instruction to help learners develop more mature writing skills. Beginning in kindergarten, teachers should make writing an integral part of math class. Not only should they ask students to write about their mathematical ideas, but they should also provide guidance to show students how to do that more effectively.

A combination of three instructional modes—showing, sharing, supporting—has proven effective for teaching students to write (see Figure 5.1). Using these modes, teachers gradually release the responsibility for mathematical writing to their students (Pearson & Gallagher, 1983).

During the showing mode of instruction, teachers are primarily responsible for writing. They provide explicit instruction through minilessons or by modeling. Students begin to assume responsibility in the sharing mode as they work jointly with teachers to write about math and then revise writing to improve it. Finally, in the supporting mode, students write independently, either individually or in collaboration with other students, as teachers monitor their work, offer feedback, and conduct conferences with them.
The three modes of writing instruction can be ongoing, sometimes occurring simultaneously, depending on the types of writing students are doing, the writing strategies they are learning, and the mathematical content they are exploring. Obviously, students need considerable support when new writing strategies are introduced, but much less support when practicing previously learned strategies. Using a variety of modes of writing instruction also allows teachers to differentiate to meet individual students’ needs (O’Connell, 2005).

Teachers can choose the level of support that best meets the immediate learning needs of their students. It is important to keep in mind, however, the strong rationale for gradually releasing responsibility to students during the learning process to help them become independent writers.

**Showing the Writing Process**

Most students only encounter mathematical writing in their textbooks. Textbook writing, though, is extremely specialized and limited. It is written for the sole purpose of making a mathematical concept or skill understandable

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**FIGURE 5.1**

**Instructional Modes for Mathematical Writing**

<table>
<thead>
<tr>
<th>Mode of Instruction</th>
<th>Primary Responsibility for Mathematical Writing</th>
<th>Kinds of Instructional Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Showing</td>
<td>Teacher</td>
<td>• Minilessons</td>
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<td></td>
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<td>• Model/think-aloud of</td>
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<td></td>
<td></td>
<td>• Model/think-aloud of</td>
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<td></td>
<td></td>
<td>revision process</td>
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<tr>
<td>Sharing</td>
<td>Teacher and students</td>
<td>• Shared mathematical writing</td>
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<tr>
<td></td>
<td></td>
<td>• Shared revision of previous</td>
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<td></td>
<td>writing</td>
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<tr>
<td>Supporting</td>
<td>Students</td>
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to students. This kind of writing typically includes definitions and multiple representations of mathematical terms and concepts; it may also describe steps in a process or offer instructions for problem solving.

Although students do that kind of writing occasionally, textbook writing is hardly a suitable model for student writers who write to justify or explain their own mathematical ideas. Students need exposure to a rich variety of mathematical writing that encourages them to think more deeply about math as they write. In the showing mode of instruction, teachers offer explicit writing instruction that shows learners how to express their mathematical ideas.

In showing students, teachers may teach a minilesson about an aspect of mathematical writing, model the process of writing a short mathematical piece, or model the revision of writing done previously. This phase of instruction is completely teacher-directed; students are primarily listeners and observers. Because of this, the lessons are short and target only one or two teaching points. Writing strategies and tips are explained to students, who are then expected to practice them in their own writing. A minilesson on a writing strategy may be presented one day and then reinforced a day or two later with modeling of the same strategy. On subsequent days, the teacher might model the revision of the piece that was written.

**Minilessons**

Minilessons are highly focused and very brief. The teacher delivers them to either a small group or a whole class of students. Each lesson is designed to provide students with a writing skill or strategy that they can apply to enhance their mathematical writing. In no more than 10 minutes, teachers deliver a teaching point and give students a bit of practice in trying what was taught or a chance to talk about what was just demonstrated, so that they will be able to apply it independently in their own writing. Calkins (2005) suggests a specific architecture, or structure, for effective minilessons, which has been adapted to facilitate mathematical writing:

1. *Making a connection.* Begin a minilesson by connecting it to what students have learned in earlier lessons, to recent mathematical experiences, or to the kinds of mathematical writing they have been doing.
Avoid making a connection by eliciting information from students. When you are the one describing the connection, it is not only more efficient, but it also enables you to inform students specifically of the connection that you have in mind when planning the lesson, rather than hoping that a student will mention it.

2. **Teaching.** Share a strategy, tool, or idea about mathematical writing that students can use to enhance their work. Explicitly tell students what you will teach them, rather than telling them what they will be doing. In planning appropriate teaching points, choose those that will develop students’ writing by addressing their current learning needs. When delivering the teaching point, both explain it and demonstrate it. Let learners know when and how they can apply what you are showing them.

3. **Actively engaging learners.** Give students an opportunity to try out what you have just taught them in a very brief guided practice session. This not only prepares students for using what they have just learned when working independently, but also serves as a formative assessment to let you know how well students understand your teaching point. Since students will have more time to apply what they have learned as they write, plan active engagement activities that are concise and focused. Consider asking students to “turn and talk” to a partner to tell about what they learned or how they will apply the demonstrated writing strategy. Alternately, have students use individual whiteboards to show what they have learned.

4. **Linking to future work.** Before concluding the minilesson, restate the teaching point and emphasize your expectation that students will use what they have just learned in the future as they write about math.

The following sample lesson scenarios offer an idea of how minilessons may be used to improve students’ writing.

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**Sample Minilesson, Grades K–2**

In this lesson, students learn how to use a math word wall as a resource when they are writing about their mathematical thinking.
Making a connection. Mathematicians, I am excited to see the writing that you have been doing. You have learned that you can share your math thinking using pictures and words. Today you are going to learn about something right here in the classroom that will make your writing easier for readers to understand.

Teaching. I am going to teach you how to use our math word wall to help you when you are trying to find the best math words to use in your writing or to spell the words correctly.

Today, I am going to write about how I plan to solve a problem. I am going to have a party and want to be sure that I have enough chairs for everyone. I know I have eight chairs, because I counted them. I have invited two families to the party. One family has three people, and the other has four people. Here is what I have written so far:

(The teacher displays a handwritten chart with the words:)

I know that I have a group of three people and a group of four people coming to my party. I have eight chairs. I have to find out how many people are coming all together so I will know if I have enough chairs.

(... and below the words includes a drawing of stick-figure people—one group of three and one group of four.)

I wrote about what I know and what I have to find out. Now I have to explain what I am going to do and why. Because I am putting groups together to find out how many, I plan to use addition to find the answer. I have two problems, though. I am not sure how to spell addition, and I can't remember the math word for the answer to an addition problem. Like all mathematicians, I want to be sure that readers understand my thinking, so I want the writing to be clear.

Let me think about this problem for a minute, and then maybe I can figure out what I can do. I remember that we have a math word wall! That will give me the information I need. Look at the word wall. There is the word “addition,” so I can spell it correctly in my writing. I also see the word “sum.” Now I remember—that’s what the answer to an addition problem is called. I can use those words as I write about what I plan to do, and it will be clear to anyone who reads it.
Whenever you need to find out how to spell a math word correctly or to find a math word that you are having trouble remembering, you can look at the math word wall.

**Actively engaging learners.** Now, let’s make believe that we are writing about a problem involving subtraction. Wow, that is a hard word to spell! We also need to know what the answer to a subtraction problem is called. Think for a few minutes without talking. How can you find out how to spell the word “subtraction” and find out what the answer to a subtraction problem is called? Now turn to a partner and share your answer.

The teacher listens to students as they talk to be sure they are using the math word wall as a resource and to identify students who still may not understand how to use the resource. After a few minutes, the students are called back together to discuss some of the ideas that were shared. Rather than calling on students at this point, the teacher shares the important ideas that were overheard, highlighting the parts of their conversations that are most important for them to remember. The students have already had an opportunity to be actively engaged. Now, the teacher can target the most significant points of their conversations, rather than having them muddled by student comments that may lack focus. It also keeps the minilesson brief, so students can spend more time writing independently after the minilesson.

As you talked, I saw so many of you point to our math word wall and then discuss how you can use it to find out how to spell math words and to find the correct math words to use in your writing. You figured out how to spell “subtraction” and what the answer to a subtraction problem is called.

**Linking to future work.** Now, mathematicians, remember that whenever you need help spelling a math word or finding the right math word, you have a valuable resource right here in the classroom. Just look to the math word wall! When we write in our math journals today, I am going to be looking to see who is using the math word wall.

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**Sample Minilesson, Grades 3–5**

The focus of this lesson is the importance of providing evidence to support ideas shared in mathematical writing.
Making a connection. In the last few days, we have been looking at some sample addition problems involving fractions with unlike denominators and writing about whether we agree with the sums that are shown. Most of the writing I have read clearly expresses whether or not you agreed. Today, I am going to teach you something very important that mathematicians do when they write about whether or not they agree with a solution or a mathematical idea. They offer evidence to support their thinking—specific reasons why they believe what they do that will convince others that their thinking is valid.

Teaching. Let’s look at this piece of student writing.

The teacher displays the following sample to the class.

In this problem, they said that when you add $\frac{1}{4}$ and $\frac{2}{3}$ you get $\frac{3}{7}$. That is wrong! They didn’t add it right. The answer should be $\frac{11}{12}$.

When I looked at this, I wondered how this student knew the answer was wrong. Nothing in the writing showed me how he knew that $\frac{3}{7}$ was incorrect.

When mathematicians write, they tell not only what they think, but also why they think the way they do and give evidence to support their ideas. So I thought to myself—what evidence could this writer give to justify this conclusion? Here is some evidence that I thought of to show that the solution was wrong.

First, I know that $\frac{2}{3}$ is greater than $\frac{1}{2}$. If I add $\frac{1}{4}$ to $\frac{2}{3}$, it has to be even greater than $\frac{1}{2}$. But $\frac{3}{7}$ is less than $\frac{1}{2}$, so the sum cannot be $\frac{3}{7}$.

Also, I know that, when you add fractions with unlike denominators, you have to find a common denominator. In this problem, it could be twelfths. The addition problem would then be $\frac{3}{12}$ plus $\frac{8}{12}$, and the sum would be $\frac{11}{12}$.

Do you see that I shared mathematical ideas to show why the sum is not $\frac{3}{7}$? I didn’t just say it was wrong without telling why I thought so. Remember, whenever you share your mathematical thinking, it is important to give the reader evidence to support what you think.

Actively engaging learners. Let’s practice. Here’s another piece of student writing.
(The teacher shows this sample to the class:)

Four people plan to evenly share a pizza that has been cut into sixths. I have to find out how much pizza each person will get. I think that if it is divided evenly, each person will get three smaller pieces of pizza, or $\frac{1}{4}$ of the pizza.

The writer did not include any evidence to justify that answer. Talk with a partner for a few minutes about the problem and about what kinds of evidence you could give to justify the solution suggested in this writing.

The teacher listens to students as they talk to be sure that they are correctly interpreting the problem and are focused on considering ways to justify the ideas in the writing. After a few minutes, the teacher calls the group back together. Rather than calling on students to share what they discussed, the teacher restates some of the important ideas mentioned by students and draws representations to make them clear to all. Students have had an opportunity to talk about evidence that could be provided. Now, the teacher wants to be sure that all students hear some of the most helpful ideas. By sharing what was overheard, the teacher keeps the lesson focused and brief.

As you talked, I heard lots of ideas about how to approach this problem. Some of you talked about what you know about fractions—that if a whole is divided into four equal parts, each of those parts will be $\frac{1}{4}$, so each person would get $\frac{1}{4}$ of the pizza. Some of you went further to say that the denominator in $\frac{1}{4}$ represents the number of equal parts into which the whole is divided, and the numerator represents the number of those parts. I also heard that there are four people, and each would get one of the parts. That also helps justify the response.

Others mentioned that the pizza-eaters would have to be creative as they divided up the pizza, since there were only six pieces. Each person could eat one of the pieces ($\frac{1}{6}$), and then two pieces would be left over. Those could each be cut in half, so each of these pieces would be $\frac{1}{12}$. Everyone would get one of them. All together, each person would get $\frac{1}{6}$ and then $\frac{1}{12}$, which added together is $\frac{3}{12}$.
Another idea I heard mentioned was that if each of the sixths could be cut in half before anyone had pizza, then each person could have three of those pieces, which would be \( \frac{3}{12} \).

**Linking to future work.** As you and your partner discussed ways to justify the solution to this problem, you were working like mathematicians. Remember the kind of thinking you were doing, and whenever you write about your mathematical ideas, provide that kind of evidence to justify them for those who read your writing.

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**Sample Minilesson, Upper Grades**

Students in this lesson learn how to brainstorm as a prewriting strategy.

**Making connections.** In the last few weeks, you learned that it is important to clearly identify ideas you are going to write about before you begin writing, so that your work is focused and not scattered. Today, I am going to teach you a prewriting strategy for thinking about what you already know about a topic before you begin writing.

**Teaching.** When I am writing, I find it helps me to have in mind many of the things I plan to say before I actually begin writing. Many mathematicians go through a prewriting process of brainstorming before they write. For example, if I am going to write about the Pythagorean theorem, I take time to think about what I know and what I might want to write about it. As I do, I jot those ideas down to help guide my writing.

(The teacher displays a piece of chart paper or a list on an interactive whiteboard on which the following terms and phrases are written:) hypotenuse, right angle, right triangle, square, triangle, equation, right triangle, formula to find the length of a side of a right triangle if the lengths of the other sides are known, and to determine if a triangle is a right triangle if the lengths of the three sides are known. (The teacher decided that having the words already written on a chart would be as effective as thinking aloud and recording them as part of the lesson, but would require less time.)

As I reflected on what I know about the Pythagorean theorem, these words and phrases came to mind, so I wrote them down. These words are giving me...
ideas about what I might write about the theorem. It takes only a few minutes, but the process of brainstorming gives me lots of ideas and some direction before I actually begin to write.

**Actively engaging learners.** I want to give you a little practice with this strategy. Working with a partner, brainstorm the words you think of having to do with the word equation. Jot down anything that comes to mind.

As students brainstorm, the teacher circulates around the classroom, listening to students’ ideas. After a few minutes, the teacher calls the class together to discuss the brainstorming process. He shares some of the ideas that he overhears, rather than calling on students. This permits him to highlight the most important ideas and keep the lesson brief and focused.

*It was wonderful to hear so much math talk going on! As I walked around the room, I heard you mention words like equality, relationships, formulas, equal signs, linear, quadratic, and much more. I even heard some of you talking about non-examples, like inequalities and expressions. It can really help your writing when you take time to brainstorm before you begin to write. The ideas you come up with help you decide what you are going to write.*

**Linking to future work.** Whenever you are getting ready to write about a mathematical topic, remember to take time to brainstorm first. Jot down some notes to guide your writing, just as mathematicians do.

In using minilessons to improve students’ mathematical writing effectively, teachers should keep in mind these recommendations (adapted from Collins 2004, pp. 27–29):

- **Limit student talk.** A minilesson is a time for teachers to provide information. Student talk during a minilesson should take place during the active engagement phase of the lesson. During the other phases, students should practice participatory listening.
- **Keep the connection brief.** The focus of a minilesson is the teaching point; the connection only sets the stage for the teaching phase. To maximize the time available for teaching and active involvement by students, the connection should be precise and to the point.
- **Explicitly state the teaching point in grade-appropriate terms.** The writing strategies being shared should be clearly stated and reiterated often.
during a lesson to ensure that students do not lose sight of what they should be learning.

- **Demonstrate the teaching point.** When students are shown *how* to apply the writing strategies being taught, they are more likely to make use of them in their own writing. Teachers can model their use, thinking aloud throughout the modeling process. After introducing a teaching point in a minilesson, a modeling session demonstrating the teaching point can be carried out on subsequent days to reinforce student understanding.

- **Incorporate familiar mathematical content in the minilesson.** Since the focus of these minilessons is on improving writing skills, demonstrations of new writing strategies should address familiar mathematical content, so that the novelty of the math does not distract from the writing strategies being taught.

- **Match the content and type of active engagement by students to the teaching point.** It is important that learners have an opportunity either to talk about their understanding of the strategies being taught or to practice using them during the active engagement section of the minilesson.

### Modeling/Thinking Aloud

For all students, including those who are attempting to put their ideas down on paper for the first time, those who are refining their general writing skills, and those who are learning to express their mathematical ideas in writing, some of the most productive learning experiences result from observing teachers as they model the writing process and speak aloud about their thinking as they do. Students not only see what is being done, but also hear what writers think about as they write. Modeling and thinking aloud by teachers demystifies the writing process, making it accessible to learners.

Teacher modeling may focus on simple topics, like helping beginning writers understand the importance of leaving spaces between words, or on more complex compositional strategies, such as the use of word banks, the logical sequencing of ideas, the justification of mathematical thinking, or any other strategies that lead students to more clearly express their mathematical thinking in writing.
The modeling of writing by teachers as they share their thinking aloud is most effective when it is teacher-directed. Students should primarily be participatory listeners as they observe the modeling. It is often best to ask students to hold any questions they may have until after the modeling is complete. Although student questions deserve answers, if they are asked during the demonstration, they may distract from the instructional focus. In addition, when the questions are pertinent, they are frequently answered as students watch and listen. Any remaining questions may be addressed after the modeling and thinking aloud have been completed.

**The writing process.** To demonstrate to students strategies for writing about mathematical thinking (noting that the modeling process should last no longer than five or ten minutes):

1. Plan your modeling/thinking aloud lesson to target an identified learning need for your class that aligns with the math standards you are teaching. Choose a writing topic to fit that need.
2. Use chart paper or an interactive whiteboard to record your writing.
3. Be specific in telling students about your writing topic, your purpose for writing, and your intended audience.
4. Inform students of any resources you will be using, such as the math word wall, instruments of measure, or textbooks.
5. Begin writing by recording your ideas on the chart paper or interactive whiteboard. Share your thinking as you write. Point out how your purpose for writing and your intended audience influence what you write.
6. Keep the writing pace brisk to maintain the attention of students, but explicitly describe the writing strategies you are modeling. Also, share how you use resources to support your writing and point out your use of any writing strategies that had been taught in earlier minilessons.
7. As you finish your writing, share your thinking about how you decided to end the piece of writing and why.
8. Encourage students to reflect and share what they learned, as well as any questions they may have about your writing process.
**The revision process.** It is important that students have opportunities to see not only the initial writing process, but also the revision of previously written work. Although not all pieces of writing call for rigorous revisions, students need to appreciate the value of rereading to improve the quality of their written product. This type of modeling is too frequently omitted. As a result, students are reluctant to revise their work or struggle with the process when they do.

To model the revision of mathematical writing:

1. *Plan a lesson to revise a piece of writing that was done previously.* This might be something that was written during an earlier modeling session or one drawn from another source. Prior to modeling its revision, decide how the writing can be improved.

2. *Explain to students how most first drafts can be made even better.* Experienced writers repeatedly reread and revise their writing. Let them know that wording may be changed, sentences added or deleted, or the organizational structure changed to make the writing more effective.

3. *Display the writing to be revised.* Read the writing together with the class.

4. *Share your reflections with students as you think aloud about the strengths of the writing and what could be improved.* Because you are modeling the process of revision, do not call upon students for suggestions—just share your thinking. Refrain from making every possible improvement; only choose those that represent a writing strategy you are teaching or that have the greatest impact on the quality of writing. Maintain a lively pace as you lead the lesson to keep students engaged.

5. *Record your revisions directly on the writing, sharing the reasoning for each change.* This is important because students are sometimes reluctant to revise their work because they do not want to “mess it up.” When they see that revision is something that writers do, they become more willing to mark up their work.
6. Ask students to reflect on the modeling session, share what they learned, and ask any questions that they may have.

7. Tell students that revision is an important step in writing well and that you expect to see them engage in thoughtful revisions as they refine their own mathematical writing.

**Effective modeling/thinking aloud.** As you implement modeling/thinking aloud sessions:

- **Tell students why you are modeling and thinking aloud.** While the purpose of this kind of instruction may seem obvious, it is not to some learners. Be explicit in letting them know that you are showing them strategies that you expect them to use in their own writing.

- **Limit student talk.** This is a time for teachers to demonstrate the process of writing and share their thinking as they decide what to write and how to write it.

- **Explicitly explain your thinking regarding what you want students to learn from the lesson.** Students often need to hear explanations repeated several times and in different ways as they begin to make sense of unfamiliar writing strategies or new ways to express mathematical thinking.

- **Be selective in what you share.** If too much is shared, students may either lose sight of the main teaching points or tune out altogether.

- **When making writing decisions, tell students why.** “Let students in on your problem-solving, ruminating, struggling writing process” (Routman, 2005, p. 148). When Routman refers to “problem solving,” she is not referring to mathematical problem solving, but instead to the process of making difficult decisions about what to write and how to write it. When you share this kind of thinking, you model the thought process with which good writers wrestle. It is important that students become aware of the thinking and decision-making processes involved in effective mathematical writing.

- **Keep the lesson brief.** Because students are listeners and not active participants in these sessions, limit their length.
• **Maintain a lively pace to hold students’ attention.** While it is important to be clear in sharing the thinking behind writing strategies that you want students to employ in their own writing, avoid getting bogged down in too many details.

• **Always include time for reflection by students as the modeling session concludes.** Encourage students to share what they noticed, what they learned, and any questions they may have.

### Sharing the Writing Process

In sharing the writing process, “teacher and students compose collaboratively, the teacher acting as expert and scribe for her apprentices as she demonstrates, guides, and negotiates the creation of meaningful text, focusing on the craft of writing, as well as the conventions” (Routman, 2005, p. 83). As they share the writing process with learners, teachers build upon what they have already modeled and provide a valuable scaffold for students who are learning to write about their mathematical thinking. With this mode of writing instruction, teachers not only share their own ideas, but also invite students to contribute to the writing process. Students assume some responsibility for writing, sharing it jointly with teachers. Because the teacher serves as a scribe, students are relieved of the responsibility for physically recording their ideas, so that they can focus on thinking about **what** to write and **how** to express it.

Teachers address specific learning needs of their students by exploring mathematical ideas and discussing possible writing strategies. With shared writing, teachers invite students to engage in writing experiences in which the entire writing process is made visible and concrete in a safe, supportive environment (“Shared Writing,” n.d.). Students gain both competence and confidence on the way to becoming independent mathematical writers. Writing collaboratively with their teachers and their peers, students practice applying newly learned writing strategies.

In shared writing, shared revision, and shared interactive writing, a teacher “expands on the students’ ideas, paraphrases their thinking, and demonstrates what cohesive writing looks like and sounds like”
(Routman, 2005, p. 85). This is a valuable teaching strategy for all writing, but especially for teaching mathematical writing, when students have so little prior experience to draw upon.

Shared Writing

Teachers and students share the task of creating a new piece of mathematical writing during shared writing. To effectively teach a shared writing lesson:

1. **Plan the lesson to target an identified learning need that directly aligns with the math standards you are teaching.** Choose a specific topic yourself or with the help of your students. Be sure the topic is of limited scope, so that it can be addressed in a reasonable amount of time or plan to break the writing task down into several sessions.

2. **Use chart paper or an interactive whiteboard to record the shared writing.**

3. **Convey to students the reasons for doing a shared writing task.** Let them know how it benefits them as mathematical writers. Discuss with students the purpose for this piece of writing and the intended audience.

4. **Solicit ideas for the opening sentences from students.** Before recording ideas, encourage all students to consider the ideas proposed and how they can be most clearly expressed. During these discussions, accept students’ ideas as you gently shape the language and structure of the writing. Model the use of precise mathematical language. Encourage the participation of all students in the writing process.

5. **Focus students’ attention on the expression of meaningful mathematical ideas, the precise use of language, and logical organization as they convert their mathematical thinking into written form.** Remember that your students are novices at composing mathematical writing and will need considerable instructional support, but avoid over-teaching. Students are easily overwhelmed when teachers attempt to address too many teaching points in one shared writing session.
6. Maintain a brisk pace to keep students fully engaged. Manage the discussion so that students have opportunities to explore a variety of writing ideas, but avoid prolonged, repetitious, or off-topic talk.

7. Pause occasionally to reread what has already been written to help students gain a sense of the logical progression of the writing and to identify any changes they would like to make to improve it.

8. If the writing is to be completed in one session, encourage students to consider how to conclude it. If it is to be worked on again at a later time, ask students to continue to think of ideas to include until they come back together to work on the composition again.

Shared Revision

Shared revision is similar to shared writing, but instead of collaborating to compose writing, teachers and students work collaboratively to improve previously written pieces. Students and teachers:

1. Choose a piece of mathematical writing that was written during a shared writing session or from another source. Plan your revision lesson to focus on a writing strategy or math concept being taught.

2. Display the first draft of the writing to be revised. Remind students that first drafts can always be improved. Review the original purpose for writing and the intended audience with students.

3. Read the writing together as a group. Ask students to share their assessment of the writing. What are its strengths? What would make it better? Help students address revisions regarding both math content and quality of writing. Solicit ideas for specific revisions. Be sure that students agree on revisions before changes are made. Record the revisions directly on the original writing. This is important because students may be reluctant to mark up their own writing.

4. Keep the pace lively to maintain student attention. Manage the discussion so that students have opportunities to explore a variety of revision ideas. Keep in mind that the end goal is not perfection, but for the students to reflect on various ways to improve the writing. The revision process
Teaching Students to Communicate Mathematically

should be driven by their analysis rather than dictated by teachers; however, teachers should spur student thinking with carefully crafted questions if students fail to recognize aspects of the writing that should be revised.

5. *Reread*, together, the revisions as they are made to ensure that they make sense and result in a more effective piece of mathematical writing.

6. *Ask* students, at the end of the session, to reflect on the revision process. Have them share what they learned and any questions they may have. Remind them that, although all mathematical writing does not need to go through a process of revision, many writings benefit from it. Let them know that you expect them to use what they have learned as they write independently.

**Interactive Writing**

Interactive writing is a form of shared writing in which teachers and students share the pen. It should only be used occasionally when there is a compelling reason to do so, because it considerably slows down the writing process. It may be used effectively to address a specific writing strategy with a small group of students who share the same learning needs or with very young students to help them gain confidence in their ability to write, including the physical act of recording their ideas.

Even when sharing the pen during interactive writing, however, teachers should be the primary scribes. Students should be asked to write only a single word or symbol at a time, rather than entire sentences.

Teachers should follow the suggested steps for shared writing and shared revision, except for calling upon students to record words, numbers, or other symbols as needed throughout the process.

**Effectively Sharing the Writing Process**

To effectively share the writing process:

- *Have clear learning goals in mind when planning a shared writing session.* Use your professional judgment to determine the most pressing instructional needs of your students in light of the math standards you
are teaching. Avoid engaging in a shared writing task for its own sake. Always have an instructional focus that addresses student needs.

- **Be flexible and willing to change the focus if the shared writing session reveals more timely and pertinent instructional needs.** Although it is certainly important to have learning goals, take advantage of valuable teaching moments if they arise. Do not, however, try to address too many teaching points in one shared writing session. In choosing to target a new learning goal, you may need to change the focus rather than just expand your goals.

- **Invite the participation of all students and value their input.** Treat the suggestions of all learners respectfully. Instead of immediately giving your reaction to an idea, ask the group to consider it. Expect students to be able to provide explanations for any suggestions that they make.

- **Encourage a productive discussion of the ideas for writing as a way of extending mathematical thinking.** The shared writing process combines the best of both oral and written mathematical communication when students engage in a lively discussion addressing math content and writing strategies during shared writing.

- **Keep the shared writing process brief.** Shared writing works best when students are fresh and actively engaged. Monitor your students as you work collaboratively with them. When you see them begin to tire, bring the shared writing lesson to a close. It can always be continued on another day.

### Supporting the Writing Process

During this phase of instruction, students write independently after progressing through a learning process in which they gradually assume greater and greater responsibility for their mathematical writing. By this time, learners should have acquired the knowledge they need to apply newly learned strategies successfully as they write on their own. Of course, experienced teachers know better than to just turn their students loose at this point; students still require plenty of support to become proficient at writing about their mathematical thinking. Teachers should continue to closely monitor and support their independent writing.
Independent Mathematical Writing

Independent writing is probably the most common form of mathematical writing that students are asked to do in school. Unfortunately, students are often asked to do this without sufficient preparation for the task. Students are more successful when they first learn what the writing process entails and the characteristics of effective mathematical writing. Mathematical writing tasks and prompts that are assigned for independent writing should allow students to practice the writing strategies and mathematical content they have been or are being taught.

By closely monitoring students’ writing progress, teachers can confirm that learners are prepared to write successfully before assigning independent writing. Being prepared, however, does not necessarily mean they will write successfully. Teachers can expect even well-prepared students to struggle as they write and practice new writing strategies; that is an important part of learning. However, being prepared means that students have a foundation of knowledge about writing and math that they can access as they write. To write well, students need ample practice with supportive coaching from their teachers.

Conferences are ideal occasions to learn more about students’ thinking as they write and to prompt them to reflect on how to express their thinking most effectively. Supportive coaching offers ongoing encouragement, as well as timely and specific feedback to students as they write. Feedback is most effective when it occurs as students write, so they have the chance to implement it in the same assignment.

Teachers’ observations as their students write about math are valuable formative assessments that can be used to identify and target additional learning needs. If many student writers are struggling with similar problems, teachers should make note of them and then give additional instruction to the whole class. For small groups of students who share the same needs, reteaching or additional instruction may be done in a small-group lesson. The needs of individual students can often be handled with one-on-one math writing conferences.
Collaborative Mathematical Writing

Collaborative mathematical writing is a form of independent writing in which two or more students work together to compose a piece of mathematical writing. The collaborative process encourages students to communicate their mathematical thinking and writing strategies orally with others, clarify their thinking, learn from one another, and work as a team. As teachers listen to their students talk, they gain valuable insight into students’ thinking.

There are drawbacks to this kind of independent writing, however. If more than two students are working together, there is an increased probability that they will stray from the task. Less assertive or less engaged students may step back from the task altogether, leaving the writing process up to others. Furthermore, collaborative writing is generally more time-consuming. Therefore, while collaborative writing does have its benefits, it is advisable to assign students to write collaboratively only on rare occasions.

Mathematical Writing Conferences

In a mathematical writing conference, teachers converse one-on-one with students about their mathematical writing (Sammons, 2014). These conversations are opportunities for students to share their thinking regarding both math and writing. Teachers sit next to students, conversing writer-to-writer. Teachers express their interest in their students’ work, ask questions to clarify their understanding of it, and try to identify logical next steps in learning for their students. They provide immediate feedback that allows students to see their writing through the eyes of a more experienced writer. Teachers are also able to use conferences to provide brief and highly focused teaching points that students can begin applying immediately.

An effective conference lasts no more than about five minutes and is composed of four parts: conducting initial research to learn more about a student’s mathematical thinking and writing process, making a decision about what is needed by the student, delivering a teaching point, and then making a link to the future writing by students (Calkins et al., 2005).
Teachers should follow these principles when conducting mathematical writing conferences:

- Keep conferences brief and conversational in tone.
- Enter a conference with a goal of discovering more about students’ thinking and instructional needs, rather than with the intent to deliver a message.
- Always include an authentic compliment before offering additional feedback.
- Offer feedback that is clear, specific, and focused on things that student writers can control.
- Provide a teaching point that directly addresses specific writing or mathematical learning needs identified in the conference.
- Celebrate the work of students as novice writers and mathematicians.

Sample Mathematical Writing Conference Scenario

In this conference, a student is responding to this question regarding pattern blocks: If a trapezoid represents $\frac{3}{2}$, which pattern block represents a whole? (derived from Thompson et al., 2008, p. 93)

Teacher: How is your writing going today, Matt?

Matt: Pretty good, I guess. I think I have answered the question, although it was kind of odd. I had to think backwards from $\frac{3}{2}$ to find out what a whole would be. It was hard to write about what I did.

Teacher: Tell me about your writing and the problems you had.

Matt: Well, I knew a whole was a parallelogram. I could just see it in my head, but I’m not sure what I wrote tells readers what I saw. This is what I wrote. See? “I know that I can put three triangles on the trapezoid. Since the trapezoid is $\frac{3}{2}$, each triangle has to be $\frac{1}{2}$. A whole must be a parallelogram.”

The teacher’s conference research indicated that Matt had a good conceptual understanding of fractional parts. He also used the correct mathematical terms for the pattern block shapes, but his written explanation was far from clear.

Teacher: You seem to understand the problem, Matt. In your writing, you used the correct math terms for the shapes of the pattern blocks. That makes it
much easier for a reader to understand your writing, but I see what you mean about it not being as clear as it could be.

The teacher decides to teach Matt to include both words and diagrams in his writing to add clarity.

**Teacher:** One thing that mathematicians often do when they communicate their thinking is use diagrams and representations to show what they mean. Can you think of a way that you could add a diagram in your writing to illustrate your thinking?

**Matt:** I guess I could draw a trapezoid and show how the three triangles fit on it. Oh, and then I could label each triangle \( \frac{1}{2} \).

**Teacher:** Yes, that would clearly show that each triangle is \( \frac{1}{2} \). Would you leave it at that?

**Matt:** No. Maybe I should draw a really dark line around the two triangles that are next to each other so somebody reading it can see the parallelogram. Let me show you.

Matt draws this diagram (see Figure 5.2).

**Matt:** Then I can write, \( \frac{1}{2} + \frac{1}{2} = 1 \). That shows that the two triangles together represent a whole. The two triangles form a parallelogram, so that represents a whole.

**Teacher:** I see now. Your diagram will show readers what you mean. I wonder if you could also describe what you did in words to make your writing more complete. Remember that when mathematicians write about math, they often include a diagram or other representation to make their writing clearer to readers. Whenever you write, keep that in mind.

---

**FIGURE 5.2**

Matt’s (Pattern Block) Diagram

\[
\frac{1}{2} + \frac{1}{2} = 1
\]
Tips for Supporting the Writing Process

To support the writing process:

- **Provide ample opportunities for students to write independently about mathematics.** To understand new writing strategies and content knowledge fully, students need much practice. They need to experience struggles as they deliberate on how to most effectively communicate their ideas. These learning experiences are the kinds that remain with and shape students as writers.

- **Ensure that mathematical writing tasks or prompts align with previously taught writing strategies and mathematical content.** Students need to practice writing independently, using their newly acquired skills and knowledge to become proficient writers.

- **Continue to reinforce writing strategies and mathematical content that has been taught.** Avoid assuming that, because writing strategies have been modeled and then used in shared writing tasks, students will use what they have learned when writing independently.

- **Have high expectations for the quality of students’ independent mathematical writing.** Routman (2005) encourages teachers to raise their expectations by sharing her experience: “children continue to surprise me with their capacity to learn and assume responsibility. I keep expecting more from them based on what I see they can do, and they rise to meet those expectations” (p. 53).

- **Only assign collaborative mathematical writing tasks or prompts when there are valid instructional reasons for doing so.** This kind of writing should be the exception rather than the rule.

- **Be selective in grouping students for collaborative writing.** Use what you know about students’ work habits, writing skills, and mathematical understanding to form productive collaborative writing groups.

- **Use your time effectively when students are writing independently.** Mingle with students as they write to get a sense of how they are doing. Closely monitor their progress, offering feedback when appropriate. Resist the temptation to use the time when students are busy writing independently to attend to “teacher tasks” unrelated to student writing.
• Maintain accurate anecdotal records based on your observations. Note areas of mastery, as well as areas where additional instruction is needed for both the class as a whole and for individual students. Use this data to plan future lessons.

What Students Need to Know About Mathematical Writing

To help students acquire a comprehensive understanding of what constitutes quality mathematical writing, teachers should teach them what good mathematical writers do as they write. Individual lessons should focus on skills that apply to writing in general, skills exclusive to mathematical writing, and skills related to the revision process. Shorter lessons with very specific, explicitly stated teaching points tend to be more effective than broader, less-defined lessons. Both during and after a lesson, students should be able to identify the teaching point.

If a teacher determines that students need to work on improving the openings of their writing, a minilesson might focus on how to write an effective opening by offering tips for composing solid openings. Students might be asked to compare sample pieces of writings to determine which opening sentences were most effective and why. The teacher might model the process of composing the opening for a piece of writing. This teaching point could then be reinforced with a shared writing lesson. As a follow-up, students might be asked to practice writing effective openings as they work independently in the supporting mode.

Figure 5.3 contains specific teaching point suggestions that may be used for instruction in any of the three modes of instruction previously described—showing, sharing, or supporting.

Teachers should be aware, though, that even when students learn these important writing skills, they still might not understand fully what good mathematical writing is. With a focus on minutiae rather than the writing as a whole, individual “trees” may well hide the “forest.” Teachers should help students develop an understanding that quality mathematical writing is more than just a list of characteristics or a description of math concepts and procedures.
Effective mathematical writers recognize that their writing has a purpose, is aimed at targeted readers, and fulfills a function—the sharing of one person’s mathematical thinking with others. It is, in essence, a transmission of information and ideas—sometimes to inform, sometimes to persuade. Students need opportunities to examine examples of mathematical writing with an eye to function. They gain a more accurate conception of what constitutes quality mathematical writing when given opportunities to read, assess, and compare multiples examples of this kind of writing.

Managing Student Writing

It is important that students write about their mathematical thinking every day, even if they just briefly reflect about the mathematical work they did that day. Many teachers find that the most convenient way to have students record their writing is in a math journal. These bound notebooks are easy for both students and teachers to manage. If student writing is done over several days, students know just where to locate their earlier work, rather than searching for
individual sheets of paper. Additionally, by having most of the mathematical writing in a journal, students, teachers, and even parents can see how students’ writing skills and mathematical knowledge have developed over time. While there may be occasions when writing is best recorded in other formats, journals are recommended for daily writing tasks. Mathematical writing can also be recorded and maintained by students on notebook paper or worksheets provided by teachers and then stored in pocket folders or three-ring binders.

With the availability of digital technology, students might try writing in digital journals or composing online blogs. The more accessible the writing is for revision and for later review, the better, though. Writing that is composed online and then forgotten is not nearly as valuable a learning tool for students.

Prewriting Strategies to Support Mathematical Writing

Prewriting tasks enhance the writing efforts of all students and are especially beneficial to struggling mathematical writers. These tasks lead students to reflect on the topic and organize their thinking prior to writing. Although prewriting tasks are more commonly used in language arts instruction and less frequently in math classes, their use by student writers leads to better products. Described below are just a few of the many easy-to-use prewriting strategies that students can apply before they write.

**Brainstorming.** Students jot down everything they think of that has to do with the topic. Once they have recorded these ideas, they highlight the ones they think are most important and plan to include in their writing.

**Sequencing.** This strategy may be used once students finish brainstorming or any other time that they have a list of ideas or steps to include in their writing. They review the ideas they plan to include and then order them in a logical sequence before they begin to write.

**Using a graphic organizer.** There are several different graphic organizers that students can use to help with brainstorming and/or organizing their thinking prior to writing. Concept maps lead students to reflect on everything they know about a concept or procedure before writing, which often results in a more complete and thorough written product. Venn diagrams are extremely helpful to writers who are comparing and contrasting mathematical concepts. Frayer models are useful in helping students define mathematical
Teaching Students to Communicate Mathematically

terms as they describe the term in words, with a nonlinguistic representation, examples, and non-examples. These are just a few of many graphic organizers that teachers can share with their students to use during prewriting.

**Analyzing a step-by-step procedure.** With this prewriting strategy, students analyze a mathematical procedure before they write its description. Something of a writing task itself, it prompts students to think through problem solving or computational procedures logically and write a brief explanation of each step. This analysis can guide students’ writing, helping them with both the sequence of the steps and descriptions of what they did during each step (see Figure 5.4).

**FIGURE 5.4**

*Step-by-Step Procedure Analysis*

In a basketball game, Kaliah’s team scored 65 points. Kaliah scored 20% of the points. How many points did she score?

<table>
<thead>
<tr>
<th>Step</th>
<th>What It Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>The team scored a total of 65 points. Kaliah scored 20% of the points. I have to find out how many points Kaliah scored.</td>
<td>I made sure I understood the information in the problem and what I need to find.</td>
</tr>
<tr>
<td>( \alpha = \text{the number of points Kaliah scored} )</td>
<td>I labeled the variable in the problem.</td>
</tr>
<tr>
<td>20% = 0.20</td>
<td>I changed the percentage into a decimal to make it easier to work with.</td>
</tr>
<tr>
<td>0.20 \times 65 = \alpha</td>
<td>To find 20% of something, I have to multiply it by 0.20. The product will tell me how many points Kaliah scored.</td>
</tr>
<tr>
<td>0.10 \times 65 = 6.5, so 0.20 = 13</td>
<td>It is easy to find 0.10 of a number. I used mental math. Then I just doubled it to find out what 0.20 of the number is. The product is 13.</td>
</tr>
<tr>
<td>0.20 \times 65 = 13 [ \alpha = \alpha = 13 \text{ points} ]</td>
<td>So, ( \alpha = 13 ) points. That is how many points Kaliah scored during the game.</td>
</tr>
</tbody>
</table>

*Source:* Thompson et al. (2008).
Writing Prompts and Questions for Student Writing

Topics for mathematical writing sometimes arise organically when interest is piqued as the students confront challenging mathematical concepts or conundrums. Teachers encourage curiosity about and interest in mathematics when they turn student questions into writing prompts that the students can use to explore topics of mathematical interest.

When students do not initiate topics, teachers can suggest prompts or questions that address the writing process or math content, promote self-reflection, or even engage students in creatively expressing their mathematical knowledge. Figure 5.5 presents mathematical prompts (most of which are open-ended so that students can address them in diverse ways) and questions that can be applied across grade levels.

Sharing Mathematical Writing

A big motivator for any kind of writing is the recognition of oneself as an author—someone who has something of value to share. Teachers increase the motivation of students to write about math when they provide opportunities for writers to share their mathematical writing with others. In addition to motivation, one of the best ways to improve students’ writing skills is to “give students a sense of authorship” (Calkins, 1994, p. 267). When they know that they are perceived as authors and that others are reading their writing, they are more willing to invest the time and effort needed to write about math well.

To create the sense of authorship, teachers may ask students to read their writing aloud to the class in an author’s chair, publish their work, or offer it to other students for peer review. With any of these options, students gain the satisfaction of being recognized as authors, sharing their own academic creations with others.

Author’s chair. Prominent in most writing workshop classrooms is an author’s chair. Seated in the author’s chair, students read their writing aloud to either the whole class or a small group of students. Listeners are encouraged to ask questions about the writing and respectfully share their feedback. Writers may request guidance from their audience regarding
### FIGURE 5.5
Mathematical Writing Prompts and Questions

<table>
<thead>
<tr>
<th>Writing About Writing</th>
<th>Writing About Math</th>
<th>Writing About Reflection</th>
<th>Writing Creatively About Math</th>
</tr>
</thead>
<tbody>
<tr>
<td>• The writing strategy I used the most today was ...</td>
<td>• Explain the steps in your problem-solving strategy. Why was it successful, or why wasn't it?</td>
<td>• I learned that...</td>
<td>• Design a bumper stick about ...</td>
</tr>
<tr>
<td>• Writing about math helps me ...</td>
<td>• In what other situations could you apply the same problem-solving approach?</td>
<td>• I understood ...</td>
<td>• Create a piece of graffiti to illustrate ...</td>
</tr>
<tr>
<td>• When I was revising my writing, I noticed ...</td>
<td>• What other strategies could you use to solve this problem?</td>
<td>• I am still thinking about ...</td>
<td>• Write about the life of a famous mathematician.</td>
</tr>
<tr>
<td>• When I write with a partner, I ...</td>
<td>• This math concept reminds me of ...</td>
<td>• I still do not understand ...</td>
<td>• If I could be any math symbol, I would be ... because ...</td>
</tr>
<tr>
<td>• What helps me the most when I am writing about math is ...</td>
<td>• I think the most efficient way to solve this problem is ...</td>
<td>• The most important thing I learned today was ...</td>
<td>• Compose a rap about ...</td>
</tr>
<tr>
<td>• I chose to use the ... graphic organizer because ...</td>
<td>• Write a problem that requires finding ...</td>
<td>• I need to improve my understanding of ...</td>
<td>• Write a newspaper headline about ...</td>
</tr>
<tr>
<td>• Using the word wall helps me ...</td>
<td>• What do you think happens if ...?</td>
<td>• What do you wonder about?</td>
<td>• Create a fairy tale about a mathematical symbol or concept.</td>
</tr>
<tr>
<td>• When I read my writing about math out loud to ..., I learned that ...</td>
<td>• Why do you think ...?</td>
<td>• What do you think is the most useful mathematical idea you have learned? Why?</td>
<td>• Write a memoir of your experience learning about a math concept.</td>
</tr>
<tr>
<td>• I justified my mathematical thinking in writing by ...</td>
<td>• Some properties of a ... are ...</td>
<td>• What helped you the most to understand ...?</td>
<td>• Write a letter to a math concept you have struggled to understand. Describe your struggles and explain what you now understand.</td>
</tr>
<tr>
<td>• I wish I knew how to ... when I am writing about math.</td>
<td>• How are ... and ... alike? How are they different?</td>
<td>• ... makes me think of ...</td>
<td>• Explain why.</td>
</tr>
<tr>
<td>• My mathematical writing is effective because ...</td>
<td>• List everything you can find out from this graph.</td>
<td>• I know ... is true. I can prove it by ...</td>
<td>• If you could meet any famous mathematician from the past or the present, who would it be and why?</td>
</tr>
<tr>
<td>• When I reread and revise my math writing, I am going to ...</td>
<td>• Draw a representation of ...</td>
<td>• What do you think is the most amazing thing about math?</td>
<td>• Write about how you decided what math strategy you used to solve a problem.</td>
</tr>
</tbody>
</table>

Source: Pearse & Walton (2011) and Thompson et al. (2008).
specific aspects of their writing that they hope to improve. Inviting writers to share their work in this way lets them know that their writing is respected and of value—that it is worth sharing with other learners.

Student authors who read their work from the author’s chair also benefit from having an authentic audience to give them feedback on their writing. Listeners learn from hearing how other students think about math, from exposure to the writing strategies they are using, and from discovering more about ways to express challenging mathematical content in writing. Listeners also learn that their feedback is valued. They listen to what is read in a more analytical way when they know that they will be asked to respond to it.

**Publishing mathematical writing.** Literacy teachers understand the worth of publishing the writing of their students. Publication is a more formal way of recognizing the work of student authors than the use of an author’s chair and, at all levels of education, publication recognizes quality writing. While publication tends to be done more frequently in the elementary grades, it is important to keep in mind the important role publication plays in college and graduate schools.

While publication can be quite an involved process, it does not have to be. The options for publication range from creating a hardcover book to simply attaching a piece of writing to card stock, bound around the edge with colorful tape. Some teachers collect student writing to compile in a parent newsletter or have their students create small “foldable” books. Also, with the availability of digital resources, student writing may be published online. Writing may also be published in the form of brochures, posters, essays, or even cartoons related to mathematical content (Routman, 2005).

Publication, of course, involves more than creating a final copy of the writing; the purpose of publication is making writing accessible to readers. For students, as for any writers, it is important that others read their writing and offer feedback. Teachers may choose to include students’ published mathematical writing in the classroom library. Students often enjoy sharing their writing with other classes or with their parents. School libraries or media centers may allow the display of published student mathematical writing for browsing by other students.
To make publication of student writing an effective part of instruction, teachers should focus on publishing several short and easily assembled works, rather than on one elaborate publication. With the frequent publication of shorter pieces of writing, students can apply newly learned writing strategies and math content in a timely manner and more quickly come to regard themselves as published authors.

**Peer review.** Another way students can share their mathematical writing is through peer review. When a student has completed a piece of writing, it is submitted to another student, who reads it and provides written feedback. The peer reader may be selected by the student or by the teacher. The peer feedback should focus on both the writing process and the mathematical content of the writing, noting specifically things an author has done well, including any questions the reviewer has, and offering constructive critical feedback. It is important that this feedback go directly to the teacher before being returned to the author, so that the teacher can ensure that comments are constructive and not hurtful. Prior to establishing peer review in a math class, it should be introduced to students with very clear guidelines as to what constitutes appropriate constructive feedback and what kinds of comments should be avoided.

**Assessing the Effectiveness of Mathematics Writing Instruction**

As you begin to incorporate mathematical writing into your math instruction, reflect on your students’ writing experiences and answer these questions:

- Do students have opportunities to learn about mathematical written communication through minilessons, modeling, shared writing, and independent writing lessons?
- Do students regularly write about their mathematical learning and thinking?
- Do students express their mathematical thinking in multiple ways, using words, symbols, and other representations?
- Do students use appropriate mathematical vocabulary in their writing?
- Do students effectively explain their mathematical thinking in their writing?
• Do students understand the importance of revision and revise selected pieces of their mathematical writing?
• Do students share their writing with others through an author’s chair, publication of their work, or peer review?
• Do students offer constructive feedback to other student writers?
• Do students assume the responsibility for assessing the quality of their own mathematical writing?
• Do students participate in mathematical writing conferences with you?

What Should I Do If...?

The following are concerns frequently voiced by teachers, followed by suggestions to address them:

• **My students write as little as possible when I ask them to write in their journals.**

  Unfortunately, some students have learned that they can get away with putting minimal effort into their writing, and so they lack motivation. Others may not clearly understand exactly what they are expected to do. Whatever the cause of the problem, you can take these steps to increase the efforts they put into their writing.

  — Maintain high expectations for what your students should accomplish and share them constantly with your students.

  — In minilessons, modeling, and shared writing, show them the kinds of writing products you expect and how they can accomplish that kind of writing.

  — Teach students specific criteria for quality writing and provide exemplars to show them what constitutes effective mathematical writing.

  — Get students excited about mathematical writing by sharing your enthusiasm for it. Let them see how intriguing mathematical ideas can be. Celebrate when you read good writing by students. Spotlight it by sharing it with the class.
— Provide timely feedback as students are working, so they can improve their writing as they work, rather than having to come back to it later for revision.

— Have students engage in assessing their own work using specific criteria for quality work.

— Vary the kinds of writing you assign. Not surprisingly, students get bored with repeatedly being asked to do the same type of writing.

• **My students will write, but are never willing to go back to revise their writing.**

This is a common problem. As with any other routine or procedure, teachers must teach students not only the value of revising their work, but also how to do it. Certainly, not all writing needs to go through a process of revision, but some should. In addition, teaching students how to revise their work positively affects their products, even as they are writing them. When learners get into the habit of considering their word usage, the organizational structure of their writing, and the mathematical content they address as they revise their writing, it carries over into their thinking whenever they are writing.

It is important to help students understand that first drafts will never be the best they can be and that all good mathematicians reread and revise their writing to ensure that it expresses clearly and exactly what they want it to say. Most importantly, students need to observe and contribute to the process of revision as it is being done.

When students have learned about the importance of revision and participated in the process of shared revision, assign a writing task for which revision will be necessary. Once everyone has completed their first drafts, provide specific time dedicated to revision. As students work, closely monitor their progress, stopping to confer with those who need encouragement or support. Ask learners to share the reasons for the revisions they are making as you confer with them. Conclude the revision work with a class debriefing session. Ask students to reflect on the process. What went well? What kinds of revisions did they make in their
writing? What kinds of issues presented problems? To conclude the ses-
sion, remind students of the importance of revision to improve writing. Provide additional support for individual students who may need it or for the entire class if they continue to be resistant to revising their writing.

- **My students only write about the steps they take in solving a math problem, but do not justify those steps.**

- **When I ask my students to write about how they solved a problem, they say that they “just did it in their head.”**

These two problems are very closely related. Students will find a solution, but then neglect to write about how they did it or explain their reasoning. In most of these situations, this problem occurs because of one of these reasons:

- *Students may not know how they found the solution or why they took the steps they did to find it.* If that is the case, it is a problem of mathematical content knowledge, rather than a writing process problem, and should be handled as such. In these situations, the writing task provides accurate formative assessment data that identified specific mathematical learning needs. Students who lack the mathematical understanding needed can hardly be expected to write about their thinking and require support that focuses on the math content with which they are working.

- *Students may be reluctant writers.* Although some learners have a strong understanding of the problem-solving process, they may not want to make the effort to explain it in writing. *If that is the case, it is a motivation problem. Solutions for this kind of problem were suggested earlier in this section.*

- *Students do not know how to express their mathematical thinking in writing.* This situation indicates potential problems with both mathematical vocabulary knowledge and writing skills. When students’ vocabulary knowledge is weak, it is difficult to clearly write about mathematical reasoning and procedures. In conferring with these students, teachers should check to see if...
insufficient mathematical vocabulary knowledge is contributing to their writing problems. If so, Chapter 6 discusses ways to build the mathematical vocabulary of learners. If vocabulary is not an issue, provide more modeling of mathematical writing strategies, opportunities for them to participate in shared writing experiences, and short writing tasks focusing on newly learned mathematical writing strategies immediately after they are taught to ensure understanding and application by students.

• **My students who are English language learners struggle to express their mathematical ideas in writing.**

English language learners face challenges in expressing themselves in English and particularly with math-specific language, so they require much teacher support. It is important to realize, though, that many native English-speaking students also struggle with mathematical terminology because they do not normally encounter it in their day-to-day lives. The strategies listed below to help English language learners can also be used with any students who struggle with mathematical vocabulary knowledge.

—*At the beginning of a writing task, be sure that these students clearly understand the question or prompt.* It is helpful to read it aloud with these students and clarify any challenging vocabulary. Consider illustrating the prompt with pictures.

—*Provide math talk cards* that have relevant math vocabulary with nonlinguistic representations for each word and sentence starters that can be used when writing about the prompt. Review these cards with students before they write.

—*If necessary, have students address the prompt or question by drawing pictures.* Later, as you confer with them, have the students talk about what they drew. Serve as a scribe and record their words to accompany their illustrations.

—*Offer authentic and specific praise* for the efforts of these learners—not just “Good thinking!” or “Good writing!” Let them know that you recognize the efforts they are making.
I have so much to teach, I just don’t have enough time to have students write about math.

Teachers are under incredible time pressures and must constantly use their professional judgment when planning lessons to balance curricular demands with their students’ learning needs. At times, though, it is important to slow down to teach more effectively. Although that may sound contradictory, slowing down can lead students to more in-depth mathematical conceptual understanding that they can apply in solving unfamiliar problems or in constructing meaning from new mathematical experiences.

While practicing a procedure repeatedly on a worksheet may help students pass a unit test composed of similar problems, learners often fail to retain the skill if they do not fully understand the math involved. As a result, the same skills must be retaught the following year, requiring even more time. Additionally, in this situation, students too often acquire little mathematical understanding that they can transfer to real-life situations. With this type of instruction, much time is spent in practice and reteaching, but long-term benefits are few.

Taking into consideration the more comprehensive learning benefits described in Chapter 4, the instructional time required for math writing is well spent. Consider these suggestions for incorporating mathematical writing into your instruction.

— When you are planning math lessons, carefully select the most important practice problems and have students complete those. In lieu of assigning all the problems, have them complete writing tasks that relate to the content of the lesson.

— Review teaching materials to be sure that each lesson aligns with the grade-level standards you are teaching. Because most math teaching resources are sold in multiple states where math standards vary, some of the lessons included may not address standards for your grade level in your state. To maximize the time you have to teach the essential standards for your state, eliminate lessons that do not align with your standards.
— *Mathematical writing tasks are ideal workstation tasks* for classrooms where math workshops are being used.

— *Incorporate mathematical writing tasks as formative assessments to determine the learning needs of your students.* With traditional assessments, students may accidentally come up with the correct solution to a problem when they do not really understand the math. Conversely, an incorrect solution without an explanation about the problem-solving process by a student does not offer the assessment information needed to allow teachers to identify crucial learning needs. The student may understand the math, but made a simple error in calculation. On the other hand, the student may lack prerequisite background knowledge or may not have understood the problem.

**Supporting Students Effectively**

To support students effectively, teachers need to know more than just whether a student’s answer was correct. When students write about their thinking, it makes it visible to their teachers. Using that information, teachers are much more likely to be able to address their students’ needs. Yet, unless students have ongoing opportunities to write about their mathematical ideas and have received instruction on how to do it effectively, assessment items requiring writing will be of little use.

**Summary**

As described in this and in earlier chapters, oral and written mathematical communication by students should be an integral part of mathematics instruction throughout all grade levels. Effective communication, however, is dependent upon a solid foundation of mathematical vocabulary knowledge, both receptive and expressive. Chapter 6 examines ways to help learners develop such a foundation.
References


Wagner, T. (2008). *The global achievement gap: Why even our best schools don’t teach the new survival skills our children need—and what we can do about it*. [Kindle 1.20.3 (47097) version]. Retrieved from Amazon.com


Laney Sammons was a classroom teacher and instructional coach for 21 years before becoming an educational consultant. Since retiring from the classroom, she has worked with teachers of all grade levels throughout the United States and Canada to help them improve their mathematics instruction. She is the author of numerous books, including *Guided Math: A Framework for Mathematics Instruction*, *Strategies for Implementing Guided Math*, *Building Mathematical Comprehension: Using Literacy Strategies to Make Meaning*, and *Implementing Guided Math: Tools for Educational Leaders*. Laney welcomes your correspondence and can be reached at Laneyas@aol.com.
Related ASCD Resources

At the time of publication, the following resources were available (ASCD stock numbers appear in parentheses):

Print Products

*Educational Leadership*: Getting Students to Mastery (December 2013/January 2014) (#114021)

*Guiding Meaningful Math Conversations* (Quick Reference Guide) by Laney Sammons (#QRG117056)

*Literacy Strategies for Improving Mathematics Instruction* by Joan M. Kenney, Euthecia Hancewic, Loretta Heuer, Diana Metsisto, and Cynthia L. Tuttle (#105137)

*Getting Started with English Language Learners* by Judie Haynes (#106048)

*Habits of Mind Across the Curriculum* by Arthur L. Costa and Bena Kallick (Eds.) (#108014)

*Succeeding with Inquiry in Science and Math Classrooms* by Jeff C. Marshall (#113008)

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