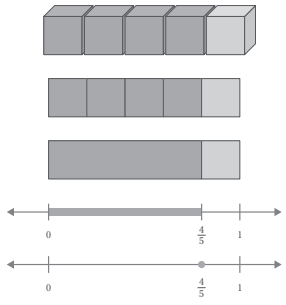
Episode 2: Visual and Tactile Representations of Fractions

FIGURE 2.8
From Concrete (3-D) to Pictorial (2-D) to Abstract (1-D)



Note: Line segments (1-D) and points (0-D) on a line are challenging abstract concepts.

Pre-Viewing Reflection

"Invest time in building meaning for fractions by using multiple concrete models and meaningful contexts," say two math educators, Kathleen Cramer and Stephanie Whitney. Indeed, what models we choose, how we plan and sequence their use, how we talk about them, and finally how we make connections among them and transition from them to the concepts discussed in Episode 1 are all critical to building a solid foundation for understanding fractions.

While "fraction circles," or equal parts of discs, are understandably the first model teachers use—building on students' experiences with slicing pies or pizzas—they are often the only model students work with for the first few years. We must take students beyond the single visualization of fractions as pizzas. No single model can capture all meanings, but multiple models, used in combination, deepen students' understanding of fractions. Each new model offers an opportunity for students to learn something new or different about the complex concept of fractions.

Reflect on the following questions before watching how Dr. Monica Neagoy leads a third grade class through an exploration of multiple representations of a fraction.

1. Are the meanings of the noun *model* and the phrase *mathematical model* clear? What about the actions of modeling in general and mathematical modeling in particular?

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2. What models (in the sense of visual or tactile representations) do you use to help your students build fraction concepts? Have you thought about their strengths and limitations?

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3. Which fraction models or representations should students experience by the time they enter middle school? Are some more difficult to grasp than others? Why is each model important?

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4. Have you ever witnessed student confusion when moving from fractional parts of a whole to fractions on a number line? What was the source of confusion? How did you remedy it?

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5. Finally, do you often think about how the words and expressions you use, the contexts or stories you select, the actions and gestures you make, and the symbols you write on the board affect students' burgeoning concept of a fraction, sometimes more than the visual or tactile model they're handling?

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Using Visual and Tactile Models

As you watch the video, observe the different models Dr. Monica uses to help students visualize a fraction and make sense of its meaning.

In the following chart, record your observations. In the middle column, jot down how you felt or what you thought when viewing and listening to each model introduced. This note could relate to any aspect of Dr. Monica's teaching or of the students' responses and comments. Then, in the right column, comment more precisely on how using the particular model may:

1. Help your students make sense of fractions,
2. Enable you to tackle misconceptions about fractions you've witnessed, or
3. Help you or your students make a connection, be it between two models or between the model and the meaning?

Model / Representation	Your Observation	Model's Potential to: • Help develop meaning • Address a misconception • Make a connection
Area model with one part shaped (different shapes named E, F, G, H)		
Area model (7 equal parts, but shaded parts not contiguous)		
Set model (a collection of 4 cubes, 2 different colors)		
Money: a concrete example from everyday life (a quarter as a fractional part of a dollar)		
A percent: another example drawn from everyday experience (the discount on an item in a sale)		
A number line model: from the length of a line segment (measure) to a unique point (modeling a number)		
A concrete commercial model: pattern blocks		
A problem-solving situation: Sara's birthday cake		

Post-Viewing Reflection

1. What specific teacher practices (actions, words, etc.) were consistent or effective throughout the lesson? What aspect of the fraction concept did each practice help develop?

2. Do you think using multiple models in the early years of fraction instruction is helpful to enhance students' understanding of fractions? Why or why not?

3. How, when, and why might you move from the area model to the number line model? A shaded section as a fraction part of a unit circle is a 2D model (area); a line segment as a fractional part of a unit segment is a 1D model (length). Reflect on the progression from "the length of the segment from 0 to the check-mark 'one-fourth' to "the point at the end of the segment representing the rational number 'one-fourth'." This progression takes several years (from grade 3 to 6). Have you ever thought about the fact that both ideas (measure and number) converge on the number line model? How might this awareness affect your future teaching of fractions?

4. What did you think of the Sara's birthday cake problem, a third grade problem containing two fractions? While you only saw the conclusion, did the paper modeling help students understand the problem and visualize the answer as a fraction of the whole cake? Might you consider using a bar model instead of a square? Why or why not?

5. How does Dr. Monica teach and reinforce the language of fractions throughout the lesson?

6. While the video was the edited version of an hour-long lesson, what would you have enjoyed seeing that was not in the video?

7. What part of the video resonated with you most? What did you not enjoy?

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Despite their importance, keep in mind that models are a means to the mathematics, not the end. Though no model is perfect, we can overcome its limitations if we constantly connect what we do in a meaningful way to the mathematics we wish to teach, embodied by the model we employ (be it a manipulative, picture, drawing, or problem situation).

Suggested Reading

Chapter 2 of Dr. Monica's book *Unpacking Fractions* (ASCD, 2017) describes the visual and tactile models used in teaching fractions to help children understand the concept. The models get more sophisticated as students progress through the grades. As in all chapters, she first identifies students' common misconceptions, then explains the underlying mathematics in depth, and finally offers challenging questions to help students tackle their misconceptions. If you don't have time to read the entire chapter, you may be interested the following passages:

- The textbox on CCSS.Math.Practice. MP5: Use appropriate tools strategically,
- The continuous models versus the discrete models,
- The Teaching Tip on Partitioning and Iterating when drawing models on the board, and
- The design of Maya's wall, a great idea for all teachers to adopt!