Episode 3: The Importance of the Unit

Pre-Viewing Reflection

A fundamental idea about fractions that must be clear to students is that a fraction does not represent a size, a quantity, or an amount in itself; it only informs us about the relationship between the part and the explicit or implicit whole. One-half of a chocolate bar does not tell us how much chocolate is involved unless we know how much chocolate is in a whole chocolate bar. Furthermore, if we don’t know the whole or unit, equality and inequality no longer hold: one-half of a whole pizza does not represent the same quantity as one-half of another, differently sized whole pizza.

Fractions have relative value, not absolute value. Teachers can help students understand this subtle notion by drawing a parallel with whole numbers. For example, when we say the number “4,” we don’t have a sense of how big, how much, or how long 4 actually is until we know what unit we’re referring to. Four miles is much longer than 4 inches!

Clearly, the ultimate goal by middle school is for all students to know and understand a fraction as a rational number, an abstract construct. But in the work leading up to this point, we frequently model a fraction in relation to a price, a length of time, a weight, a length, an area, a volume, a collection of items, and so on—in short, a quantitative whole. Therefore, to know the quantity I am talking about when I say “two-thirds,” I first need to know the quantity represented by the whole or unit.

Reflect on the following questions before watching how Dr. Monica Neagoy prompts third grade students to appreciate the importance of the unit when working with fractions.

1. In the U.S., depending on the student textbooks or teacher resources, the terms whole, unit, and one are often used interchangeably. Which do you use and why? Can you think of a rationale for starting with “whole,” then adopting the word “unit,” and finally using the term “one”?

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2. Think about the questions you pose in class and the problems or tasks you create for assessments:
   - Do you start with the whole / unit and move to the part(s), or do you also ask to determine the whole / unit knowing the part(s)?
   - Is the whole routinely made explicit, or is it left implicit?

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3. Have you openly addressed in class the importance of the whole / unit in working with fractions when teaching equivalence, comparison, computation, and so on?
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4. Do you find that many students think of the whole as “one thing”? For example, if you are modeling fractions with pattern blocks, do your students prefer the yellow hexagon for the whole? How might you remedy this misconception?
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5. Do many students think of “a fraction on the number line” as a fraction living between 0 and 1? Or, if not, do they have more difficulty conceptualizing a fraction greater than 1? If so, what might be some reasons for the difficulty?
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6. How would you respond to a student who says that the drawing below is a pictorial model for the fraction three-fourths?
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### Focusing on the Unit

As you watch the video, take note of the mathematical discourse and the examples Dr. Monica crafts to help students focus on the unit before answering the fraction question.

In the following chart, record your observations. In the middle column, jot down how you felt or what you thought when viewing and listening to each element selected from the video. This note could relate to an aspect of Dr. Monica's teaching or what it prompted in students' thinking or your own learning. Then, in the right column, comment more precisely on how the question or comment addressed the goal of the lesson: becoming more mindful of the importance of the unit.

<table>
<thead>
<tr>
<th>Model / Discussion</th>
<th>Your Observation</th>
<th>Developing the Concept of the Whole or Unit</th>
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</thead>
<tbody>
<tr>
<td>“What do you suppose a unit fraction means?”</td>
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<tr>
<td>“How would I make ( \frac{1}{5} ) or ( \frac{1}{23} ) (of a pizza, on a number line or in any representation)?”</td>
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<tr>
<td>“Nobody is saying the key word: equal parts!”</td>
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<tr>
<td>“What fraction is this (green pattern block triangle)? Put your thinking caps on: this is a thinking question!”</td>
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<tr>
<td>[Referring to the previous question] “Did I say what the whole or the unit was?”</td>
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<tr>
<td>“Now tell me what the unit would be if I told you the green piece represents ( \frac{1}{2} )?”</td>
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<td></td>
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<tr>
<td>[Concluding] “So it doesn’t make sense to ask this question, ‘what fraction is this green piece?’”</td>
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</tbody>
</table>
“How do we know that this (green pattern block triangle on the red pattern block trapezoid) represents \( \frac{1}{3} \)?”

“Think of a creative shape that you could make of which the green triangle is \( \frac{1}{12} \).”

Post-Viewing Reflection

1. Might your students benefit from a similar exploration of the fractional value of the green pattern block piece? Would it help them understand that one piece, the green triangle, can represent a variety of fractions? For example, the fraction one-half (if the whole is the blue rhombus); the fraction one-third (if the whole is the red trapezoid); the fraction one-fourth (if the whole is two rhombi); and so on.

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2. Was the concrete model selected for this sequence effective in helping third grade students appreciate the necessity of knowing the whole before determining the fractional value of a part?

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3. Dr. Monica often warns, “Beware of one-way mathematics!” Is it your experience that most questions students field about fractions are whole-to-part questions? Meaning, students are asked to find the fraction(s) given the implicit or explicit whole of unit? What impact do you think a part-to-whole question, such as “make a whole of which the green is one-twelfth?”, might have on students? List at least three ideas.

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4. What ideas will you take away from this video sequence to use with your students?

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Suggested Reading

Chapter 3 of Dr. Monica’s book, *Unpacking Fractions* (ASCD, 2017), addresses the importance of the unit when working with fractions. As in all chapters, she first relates a fraction lesson on the subject (with student and teacher discourse), then identifies students’ common misconceptions, explains the underlying mathematics in depth, and finally offers challenging questions to help students tackle their misconceptions.

If you don’t have time to read the entire chapter, you may be interested the following passages:

- Ed’s story: A fourth grade teacher whose student Tyler helped him understand the importance of clearly stating the unit,
- Working with a Variety of Units, and
- The two problem sets are rich with ideas that target student thinking versus just doing.