ASCD STUDY GUIDE
UNPACKING FRACTIONS

Episode 4: Teaching the Concept of Equivalence—Not Just the Rule

Pre-Viewing Reflection

Equivalence is a critical concept of mathematics, one that accompanies students throughout their mathematical journey. Examples of equivalence include equivalent notations for numbers, equivalent fractions, equivalent expressions, equivalent equations, equivalent functions, and equivalent sets. Note that defining equivalence depends on the mathematics objects involved: for example, two equivalent sets may have different elements but must have the same cardinality [A = {1, 2, 3} and B = [X, Y, Z}], whereas two equivalent fractions may be written with different symbols but must represent the same value [$\frac{1}{4} = \frac{3}{12}$].

Fraction equivalence is poorly understood, yet it is a foundational concept for comparing, ordering, and operating with fractions. You may recall this often-cited statistic: in the 1990s, in a NAEP mathematics assessment item, approximately one out of three eighth graders had difficulty shading areas or drawing pictures to model equivalent fractions and, conversely, selecting an equivalent section for a given pictorial model. To prevent such outcomes for our students today, we must plant the seeds of "understanding equivalence" early on.

In our era of National Council of Teachers of Mathematics standards and Common Core State Standards for Mathematics (CCSSM), teachers are expected to cultivate meaning for fraction equivalence starting in third grade. Yet too often, the fraction algorithm is introduced too soon. It is common to see this type of diagram, by way of "an explanation," written on the backboard of an elementary classroom:

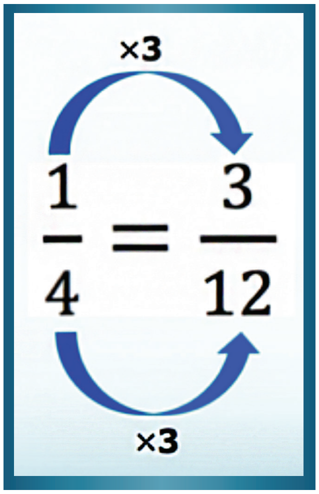

Figure 1: Diagram Illustrating the Equivalent Fraction Algorithm

Reflect on the following questions before watching how Dr. Monica Neagoy introduces the notion of fraction equivalence to two different groups of students, using different models.

1. How do you introduce the notion of equivalent fractions in the early stages of mathematical learning? What type of models and modeling do you use?

2. While the CCSSM suggests explaining fraction equivalence by using visual fraction models, do you find yourself reverting to the numerical relationships among the numbers within the equivalent fractions at hand, as in Figure 1?

3. More precisely, do you find yourself providing the following common explanation to third or fourth graders: "Multiplying the numerator and the denominator of the fraction one-fourth by three amounts to multiplying the fraction by three-thirds. But three-thirds equals one, and multiplying any number by one doesn't change it. Consequently, one-fourth and three-twelves are equivalent"?

4. Ponder the following facts: the explanation in bullet 3 presupposes the mastery of three concepts:
   - That $^{n}/_{n} = 1$ for any value n;
   - That 1 is the identity element for multiplication; and
   - That $(^{a}/_{b}) \times (^{c}/_{d}) = (^{ac}/_{bd})$, the general algorithm for multiplying fractions

   Yet these concepts are taught and solidified in grades 5 and upward.

5. When showing two equivalent fractions to fifth and sixth graders who know equivalent fraction algorithm (EFA; Figure 2a), do you ever highlight the within ratio (Figure 2b), or only the between ratio (Figure 1)?

$$\frac{a}{b} \text{ is equivalent to } \frac{(n \times a)}{(n \times b)}$$

Figure 2a. Equivalent Fraction Algorithm (EFA)

$$\times 4 \quad \frac{1}{4} = \frac{3}{12} \quad \times 4$$

Figure 2b. The "Within Ratio" of Two Equivalent Fractions

6. Returning to the example in Figure 1, young students have difficulty accepting that the second fraction is not "3 times bigger than the first." What explanations, visuals, contexts, or stories do you provide to convince them?

7. Finally, the language we use in our teaching affects students' burgeoning concept development. Do you ever use the verb "to reduce" when explaining the action of simplifying a fraction, say three-twelves, to show it is equal to another, say one-fourth? If so, can you see how the verb may be misleading for young students?

Teaching the Concept of Equivalence—Not Just the Rule

As you watch the video, make a list of the different ways Dr. Monica introduces the concept of fraction equivalence. Comment on a couple of strategies and describe how you think they may help in building understanding. Consider recording questions, explanations, student responses, and reactions to student responses, as part of your notes.

Strategy	Developing the Concept of Fraction Equivalence

Post-Viewing Reflection

1. Comment on the use of visuals in this video sequence. How important is visualization in aiding students' understanding of equivalent fractions?

2. When using tactile or visual examples for teaching equivalence, the preferred model is the area model. What did you think of the discrete example used (a collection of objects: 3 out of 12 were red and 9 out of 12 were blue). Might you use such an example next time you teach the meaning of equivalence? Why or why not?

3. What explicit instruction was instrumental in helping students understand the meaning of equivalent fractions?

4. What did you think of the student's answer, "($\frac{8}{16}$) = ($\frac{16}{32}$)"? Was that student on the road to figuring out the algorithm, meaning the generalized formula? Explain.

5. Many students who "know" the EFA (Figure 2a), will nevertheless fill in the following blank with "13." What misconception does the answer reveal? Having watched the video, how might you address this misconception?

$$\frac{2}{5} = \frac{10}{?}$$

6. At the end of the video, Dr. Monica mentions the progression from concrete to pictorial to abstract for learning concepts. Do you espouse this theory, a staple of the Singaporean approach to teaching mathematics, adapted from Jerome Bruner, of the three stages of learning: enactive, iconic, and symbolic?

Suggested Reading

Chapter 4 of Dr. Monica's book *Unpacking Fractions* (ASCD, 2017) addresses the teaching of fraction equivalence with understanding, not just imparting the rule. As in all chapters, she first relates a fraction lesson on the subject (with student and teacher discourse), then identifies students' common misconceptions, explains the underlying mathematics in depth, and finally offers challenging questions to help students tackle their misconceptions.

If you don't have time to read the entire chapter, you may be interested in the following sections:

- Recognizing Misconceptions, and
- Unpacking the Mathematical Thinking.