Episode 5: Comparing and Ordering Fractions

Pre-Viewing Reflection

A fundamental property of our number system is that given any two numbers, a and b, one of the following three cases is true:

$a < b$ $\quad$ $a = b$ $\quad$ $a > b$

To compare two quantities or numbers judiciously (i.e., to figure out which of the three cases above is true), one must first have good number sense—and an important part of number sense is recognizing the relative magnitude of numbers. When it comes to fractions, which are special kinds of numbers (positive rational numbers), research shows and classroom practice confirms that this ability is direly lacking. An often-cited test item from a NAEP exam once asked eighth graders to approximate the sum of $12\frac{}{}/13 + 7/8$ without computing it and to select the correct answer from among five choices. Over 50 percent of the students tested chose 19 or 21 as the best estimate of the sum. The fourth graders you will see in this video would probably be able to say—and justify—that because both fractions are close to the benchmark 1, their sum is close to 2.

As you saw with fraction equivalence, or will see with fraction computation, many students can't compare two fractions without resorting to a memorized rule. Once again, the reason for this unfortunate state is that rules for comparing fractions are taught too soon. Students learn to rely on the mechanistic application of the rules rather than on their mathematical reasoning to decide which of two fractions is greater, or if they are equal, and why.

By fifth grade, students have developed a pretty good understanding of a fraction as a single number, represented by a unique point of the number line. The lesson you are about to see introduces three strategies for comparing two fractions that students will be learning during their upcoming school year. The three strategies constitute the basic strategies all students should know well and be able to apply effectively by fifth grade.

UNPACKING FRACTIONS

Reflect on the following questions before watching how Dr. Monica Neagoy encourages a group of fourth grade students to use their fraction number sense to decide which of two fractions is greater.

1. Can you imagine a situation in which one-third of a _____ is greater than one-half of another _____? Yet we know from experience that $\frac{1}{3} < \frac{1}{2}$, so what underlying assumption do we make when comparing two fractions? Do you routinely make this assumption explicit to students?

2. What three basic strategies should students in third, fourth, and fifth grades know and understand well before entering middle school?

3. Do you know and teach other strategies for comparing fractions? If so, describe them.

4. Besides describing the numerator as "the number above the fraction line" and the denominator is "the number below the fraction line," what conceptual understanding of these two numbers do you foster in your students? Do your explanations ever involve the role of "divisor" for the denominator and "multiplier" for the numerator?

5. How does understanding these meanings of numerator and denominator contribute to understanding the magnitude of a fraction, and therefore to the ability of comparing two fractions?

6. What models are useful for comparing and ordering fractions with common as well as different denominators? How many different models do you encourage your students to use?

Comparing and Ordering Fractions with Meaning

As you watch the video, record the attitude, questions, and discourse Dr. Monica models to help students compare fractions. Jot down what strikes you most in terms of the teachers' actions and discourse and in terms of students' responses, thinking, and doing. You will see that the lesson addresses some generalizations, assumptions, and misconceptions. During this lesson (of which about 40 percent was captured on video), do you think some students made progress in their understanding of how to reason about the magnitude of fractions?

In the following chart, note any general or specific observations not necessarily related to fraction comparison. For instance: your reaction to how Dr. Monica explains that while she knows what she wants to teach for the day, she "goes with the flow" and lets students' ideas, reactions, questions, and answers guide her and "carpe momentum."

Strategy or Task	Teachers' Actions and Discourse (question / strategy / model / words /etc.)	Students' Thinking, Saying, and Doing (indications of concept development)
Comparing fractions with common numerators		
Comparing fractions with common denominators		
Comparing fractions to a benchmark		
Comparing fractions with different numerators and different denominators		
Other Observations and Reflections		

Post-Viewing Reflection

1. How do students use their understanding of the whole, the numerator, and the denominator to make sense of comparing two fractions?

2. What did you think of the strategies you saw in the video? Were they effective in helping students understand the questions and figure out the answers? Do you discuss these same strategies with your students?

3. What role does fraction equivalence play in understanding the three strategies?

4. Did you see evidence of student understanding? If so, give two or three examples.

5. Did watching the video make you more mindful of your own teaching? Might you do something differently next time you teach a lesson about fraction comparison? If so, give an example or two.

6. Regarding models used by teachers and students to illustrate fraction comparison, do you also use fraction circles (more accurately called disc sectors), connecting cubes, tape diagrams, number lines, and double number lines? Which did you particularly like in the video, and why?

7. Reflect on using this introduction next time you begin a lesson on comparing fractions:

   Hold up an opaque bag of chocolate bars and tell students that the bag contains some whole bars and some half bars of the same brand of chocolate. Then ask students to tell, by a show of hands, which bar they want: a whole bar or a half bar. Then the moment of truth: pull out the *whole miniature bars* followed by the *half giant-size bars*.

   What point does this story drive home?

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Suggested Reading

Chapter 5 of Dr. Monica's book *Unpacking Fractions* (ASCD, 2017) discusses the clever strategies students learn in elementary school to compare fractions. As in all chapters, she first relates a fraction lesson on the subject (with student and teacher discourse), then identifies students' common misconceptions, explains the underlying mathematics in depth, and finally offers challenging questions to help students tackle their misconceptions.

Chapter 5 is full of mathematical ideas and practical tips that every teacher will appreciate and can apply. In particular, you will find the answers to all questions posed above, and much more. If you don't have time to read the entire chapter, you may be interested in the following sections:

- Recognizing Misconceptions,
- Unpacking the Mathematical Thinking,
- The additional strategies for comparing fractions not seen in this video, and
- The Teaching Tip: "Focus strategy rather than the answer."