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The Teacher's Role in Increasing Student Understanding of Mathematics

When students discover mathematics concepts for themselves and refine problem-solving skills in small groups, they learn mathematics *and* self-reliance.

Photographs by Michael Zide

Teacher: What happened? Why did you do so poorly on the homework problems?

Student: Gosh! I don't know. I understood the problems in class when you were going over how to do them. But when I got home and tried to do them myself, they just didn't make sense.

Most mathematics teachers are familiar with this scenario. It reflects a well-documented lack of conceptual understanding and problem-solving skills among mathematics students in the United States (NAEP 1983). To survive in mathematics courses, many students attempt to compensate for their lack of understanding by memorizing mathematical procedures and formulas (Simon 1985).

To address this problem, mathematics teachers increasingly are adopting two teaching strategies: the exploration/discovery approach to mathematical concepts and the use of pair and group problem solving (Finkel and Monk 1985, Whimbey and Lochhead 1980). These strategies require teachers to shift from their traditional role as lecture-demonstrators to a role that demands new skills in planning and facilitating student work.

At SummerMath for Teachers, Mount Holyoke College's inservice training program, elementary and secondary school teachers experience the effect of these strategies on their own learning. They develop an enthusiasm for the depth of understanding that these strategies engender and for the problem-solving skills they develop. When teachers return to their own classrooms, they begin the difficult task of transforming their role in order to use these strategies effectively. A major part of the SummerMath experience, therefore, is the development of new or under-used teacher skills that are essential in this new role.

For example, in a public junior high school classroom in South Hadley, Massachusetts, students enter a mathematics classroom and arrange themselves in groups of three or four. The teacher, Marie Appleby, has arranged on tables around the room materials for today's work, which involves ratio and proportion. She gives each student a worksheet outlining a sequence of tasks for the students to perform cooperatively. The room quickly becomes alive with students experimenting, sharing ideas, organizing data, and drawing conclusions. At times students engage in animated debates.

Students in Marie Appleby's 8th grade mathematics class take measurements to compare the diameter and circumference of circular objects (to discover the ratio represented by pi).

Halfway through the class, each group of students receives a blank overhead transparency on which to summarize their findings and solution methods. These are used in reporting their group's conclusions in the class discussion that follows.

Discovering Mathematical Concepts

Students who are exploring and discovering new concepts for themselves have the opportunity actually *to do* mathematics rather than passively learn about mathematics (Papert 1972). The discovery approach challenges students to think more deeply about the concept and to create representations and explanations of it that connect with their prior experience in a personally meaningful way. As a result, they retain their understanding of the concept longer than students who have only the teacher's or the textbook's explanation (Berman 1982).

During the late 1960s and early 1970s, a great deal of instructional experimentation took place under the banner of discovery learning. Too often, however, discovery learning became "leave the student alone and he will discover what he needs to know." Needless to say, that approach was often disastrous. Left alone, students are not apt to make mathematical discoveries that took the best mathematicians in history thousands of years to discover.

Facilitating students' discovery of mathematical concepts requires far more than a laissez-faire approach in which the teacher does little other than stay out of the way. Rather, this difficult and demanding form of teaching requires teachers to examine the cognitive structure of the concepts to be taught and then to create series of experiences that will offer students the opportunity to explore the domain and discover these concepts.

Pair and Group Problem Solving

Student discovery of mathematical concepts is further enhanced by our second strategy, pair and group problem solving. Cooperative learning situations in which students work together in pairs or small groups traditionally have not been a part of mathematics classes. Now, however, these situations are regarded as an essential part of the mathematics learning experience and one that max-

Steve Lamm and Merry Sue Ahlgren pair problem-solve during the SummerMath program for Teachers, experiencing the kind of cooperative and focused work they will plan for students in classes they teach.

imizes the benefits of student exploration and discovery (Narode 1985, Johnson and Johnson 1979).

Two important benefits result from pair and group problem solving. First, students are exposed to diverse thinking and problem-solving approaches. Second, they develop metacognitive skills—knowledge about their own cognitive processes and the ability to use them. Metacognition, which until recently received little attention in mathematics instruction, is knowledge about the use and limitations of particular information and strategies and the ability to monitor and evaluate their use (Flavell 1976, Schoenfeld 1985). These skills are variously called executive, managerial, control, or metacognitive skills.

These terms indicate that having a storehouse of knowledge and strategies is useless without the ability to select appropriately from that storehouse and to organize and monitor work on a task. Unfortunately, instruction often focuses on expanding the storehouse without developing the skills for managing it.

A student who is working alone is less likely to become aware of managerial or metacognitive processes in problem solving. He may seize on the first strategy that comes to mind without reflecting on its merits and limitations. In contrast, students working together on a problem that is not routine for them (for which they do not possess a solution strategy at the outset) are faced with the tasks of generating possible solution strategies, selecting one to try first, deciding how long to stay with that strategy, and determining what strategy to try next based on what has been learned. By explaining one's ideas to the other group members and helping the group decide on the merits of different ideas, a student develops the metacognitive aspects of problem solving.

The Role of the Teacher

Teaching mathematics through discovery and pair and small-group problem solving requires the teacher to give up the role of imparter of information and to become the architect and facilitator of active learning. It is the skill of the teacher in planning lessons and encouraging and eliciting student thought that determines the success of this approach.

In planning discovery lessons, the teacher has several important roles.

1. *Identify and prioritize what needs to be learned*. It is not sufficient to "cover" the textbook curriculum. Informed by research and his or her own professional experience, the teacher must decide what is most important for students to learn. The time, effort, commitment, and creativity required to implement these new strategies should be directed to the most important parts of the curriculum. Noncontent-specific goals—developing problem-solving and metacognitive skills, developing creativity, and increasing math confidence—also should be considered in establishing priorities.

2. *Distinguish between facts, procedures, and concepts*. Lecture-demonstration may be effective in teaching facts and procedures that can be mastered through memorization, imitation, and repetition. However, the learning of concepts is often better served by the exploration/discovery approach, which allows students to build upon their previous knowledge and to create a personally meaningful understanding of the concept.

3. *Organize concepts hierarchically*. The understanding of concepts (and often the retention of facts and procedures) requires prerequisite understandings. In discovery learning, students must be able to use previously learned concepts and strategies in new ways and new combinations, abilities that require thorough understanding beyond mere exposure to previous material. If an essential part of the foundation is weak or missing, discovery breaks down. Therefore, discovery learning demands particular attention to sequential planning.

4. *Divide what is to be learned into appropriate increments*. The emphasis shifts from how much the teacher can cover, or even how much ought to be covered, to how great a step the students can negotiate. Students should be challenged to increase their understanding, but the challenge should remain within their grasp.

5. *Create or adapt activities that stimulate the development of the desired concept*. Considerable effort and imagination are required to develop the activities that allow students to discover important concepts. In creating these activities, the teacher faces the question of how structured to make them. To allow students to develop their creativity, it is often advantageous to use less structured activities first, and then gradually to increase their focus and structure. Students who discover the concepts early on can proceed to more advanced tasks, while more structured activities enable other students to discover the key concepts with more guidance. Ideally, students receive only as much structure as they need.

Judgments about mathematics learning and teaching of the type that I have described are seldom exercised by teachers who simply "cover a textbook." By contrast, teachers who choose to involve students in pair and group problem solving and in discovery learning can assist them in achieving a deeper understanding of mathematics and the processes through which they can learn.

In implementing this process, the teacher has three particularly critical roles.

1. *Ask questions that encourage students to reflect on and verbalize their thought processes.* Such questions focus students' attention on metacognition, helping them become aware of thought processes that would otherwise go unnoticed. Inquiring about and listening to students' thinking strategies, teachers demonstrate that they value careful thinking and diverse strategies for solving problems. In this role, teachers skillfully deflect students' inquiries of "Is this the correct answer?" Instead, teachers encourage students to evaluate their own work.

2. *Provide subtasks where needed.* No matter how thoughtfully the teacher prepares the lesson, some students will be unable to solve the problems or discover the concept being presented. For them, the teacher may need to divide the original problem into subtasks that still involve learning through personal discovery. After working on the subtasks, students often will be able to capitalize on the new learnings to solve the previously insurmountable problem.

3. *Regularly evaluate the students' understanding.* In order to plan subsequent lessons, it is important for the teacher to assess what students understand through conversations with small groups of students as well as through assignments and tests. If a concept has a high priority or if it will be important to the discovery of more advanced concepts, students must understand it thoroughly before they move on to new work.

Students have come to believe that what is really important is what is on the test. If thinking processes, creativity, and divergent thinking are valued, teachers must base an important part of tests and grades on performance in these areas.

New Life in the Classroom

In order for students to use mathematics with confidence, they must have opportunities to learn actively—discovering concepts and communicating their thinking to their peers. Building on an understanding of cognitive processes, the teacher is the architect and facilitator of this approach, infusing new life into the classroom and enabling students to become more powerful problem solvers.□

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